Comparing performance on chaos control via adaptive output-feedback

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Abstract

Performance of four controllers is experimentally compared and evaluated in context of chaos suppression. Four output-feedback controllers are used in experiments for comparison. First three schemes utilize an adaptive observer to estimate the states and parameter required for feeding back and with different techniques, which are: (i) feedback linearization, (ii) backstepping, and (iii) sliding mode. The fourth scheme is a (low-parameterized) robust adaptive feedback. A simple class of dynamical systems that exhibit chaotic behavior, called P-class, is considered as benchmark due to involves distinct chaotic systems. The need of comparison is motivated to ask: *What is the suitable adaptive scheme to suppress chaos in an specific implementation?* Results show a trend on different applications, are illustrated experimentally by means circuits, and are discussed in terms of control effort. This comparative study is important to select a feedback scheme in specific implementations; for example, synchronization of complex networks.

Keywords: Chaos suppression, Adaptive observers, Controlled nonlinear dynamics.

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1 Introduction

Chaos control comprises both problems suppression and synchronization of chaotic systems and can be potentially exploited to deal with current engineering problems like, among others, chaos suppression on dc-dc converters; use of multimode laser in surgery of carbon nanotubes; regulation of fluid dynamics; design of systems for secure communications via internet; and some problems in biomedical sciences as arrhythmias or epilepsy. Particularly, chaos suppression problem has been a studied topic to address these problems (see [1, 2, 3, 4]). Complementarily, synchronization problem has been exploited to deal with secure communication, synchrony on multimode lasers, biological systems and, more recently, complex networks (see [5, 6]). Since a wide variety of applications, diverse schemes have been proposed to understand mechanisms of the chaos control [2] or to design control devices [4, 7].

Major concern in the study on chaos control is about the possibility of driving a desired chaotic behavior along time (i.e., to induce a desired behavior on dynamical systems via feedback interconnection). For instance, several control schemes have been widely studied in last two decades to induce synchronous behavior (see [4, 6, 8]). As matter of fact, a practical implementation is often limited by the information available for feeding back. Then, a practitioner has only partial information available to control; i.e., only measured states or nominal parameters values. This fact has served as motivation to exploit diverse adaptive control techniques like, among others, feedback output linearization, backstepping, sliding-mode, and observer-based. These approaches allow diverse schemes that lead us to the following problem: to select a specific suppression scheme in searching a desired control performance. This problem is a challenge if we consider the trade off among simplicity in implementation, control effort and convergence rate. This particular problem takes major relevance in light of recent results; for example, on stabilization of complex networks [6]. Moreover, due to chaotic dynamic is highly sensitive to initial conditions and parameter values, chaos control has been oriented on robust feedback approaches [4, 5]. Most of above schemes are robust in face to model uncertainties and to ensure convergence and stability. Thus, selection of a specific technique can depend on: performance of control, structural simplicity, and closed-loop stability, etcetera.

The purpose of this paper is to study performance in suppressing chaos by stabilization via feedback control. In order to reach this purpose, a class of chaotic systems is considered as benchmark to evaluate the controllers performance. The considered class includes the most important chaotic systems in the form $\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})u$ with $\mathbf{x} \in \mathbb{R}^3$ and vector fields $f, g : \mathbb{D} \to \mathbb{R}^3$ have, at least, first derivative, where $\mathbb{D} \subseteq \mathbb{R}^3$ stands for the system domain. Particularly, the paper is focussed on chaotic systems with vector fields f, g that can be transformed to the jerky form. The performance study is done as a comparison of four controllers and is carried out experimentally in electronic circuits for best performance of each compared controller. That is, control parameters were tuned such that each controller had the least effort and induced the fastest convergence. The discussion is presented in terms of the quantitative performance measured by three performance indexes.

Paper is organized as follows. A class of chaotic systems is presented in Section 2. Section 3 describes three controllers for chaos suppression: state feedback linearization, backstepping techniques, sliding-mode. These three controllers are based on an adaptive observer designed to estimate unmeasurable state and unknown parameters. Although these schemes were reported in [4, 5, 13], in seek of completeness, closed-loop stability analysis is included. Section 4 shows the fourth scheme, which is a low-order parametrized controller. In this contribution, the main point is performance comparison for the four controllers under noise and parametric uncertainties. Such controllers are designed in Section 5 for the P-class presented in Section 2. The performance comparison is carried out experimentally on the realization of chaotic system in an electronic circuit. Finally, Concluding remarks are given in Section 6.

2 Chaotic Systems of P-Class

A class of systems is introduced in this section. This class contains dissipative chaotic terms and can be transformed into the equation $\dddot{x} = J_P(x, \dot{x}, \ddot{x})$, which is defined by a polynomial jerk function (for details see [9]). Furthermore, jerk equation may exhibit chaotic behavior for a set of parameter values. Jerk equation represents different nature or man-made systems and is a sub-class of Lur'e systems when bursting in velocity is involved (see [10] and references therein). Without control, jerk equation has the homogeneous form

$$\dddot{x} + \alpha\ddot{x} + x - \phi_Q(x, \dot{x}) = 0 \quad (1)$$

where $\phi_Q(x, \dot{x})$ includes quadratic terms in state and its derivatives; e.g., $\phi_Q(x, \dot{x}) = \dot{x}^2$ or $\phi_Q(x, \dot{x}) = x\dot{x}$.

In what follows, by defining a coordinates change as $x_1 = x$, $x_2 = \dot{x}$, and $x_3 = \ddot{x}$, the equation (1) represents the state space form: $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = -x_1 - \alpha x_3 + \phi_Q(x_1, x_2)$. Thus, equation (1) constitutes a family of systems with five monomials on its right-hand side, four coefficients can be ± 1 by re-scaling state variables and time, and a unique parameter related to damping $\alpha > 0$. Then, among many others, the following collection of dynamical systems is included in this family [9]:

$$\Sigma_1 : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -\alpha x_3 - x_1 + x_2^2 \end{cases} \qquad \Sigma_2 : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -\alpha x_3 - x_1 + x_1 x_2 \end{cases}$$
$$\Sigma_3 : \begin{cases} \dot{x}_1 &= x_2 + 1 \\ \dot{x}_2 &= -\alpha x_2 + x_3 \\ \dot{x}_3 &= x_1 x_2 \end{cases} \qquad \Sigma_4 : \begin{cases} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_1 + 1 \\ \dot{x}_3 &= -\alpha x_3 + x_1 x_2 \end{cases} \quad (2)$$
$$\Sigma_5 : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\alpha x_2 + x_3 \\ \dot{x}_3 &= -x_1 + x_2^2 \end{cases} \qquad \Sigma_6 : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\alpha x_2 + x_3 \\ \dot{x}_3 &= -x_1 + x_1 x_2 \end{cases}$$

All systems in collection (2) is grouped in a class by using $\mathcal{C}^k$-equivalence of vector fields [9, 11].

Noted that the P-class can be found for a large variety of systems (see Appendix). As matter of fact, for example, there exists conditions such that the Rössler, Lorenz, and Chen systems can be written as the form (2). Additionally, dynamical properties can be studied through transformations of systems; as, for example, its stability can be analyzed via preservation [12].

3 Observer adaptive control schemes

Among diverse techniques used to control, we can mention state feedback linearization, backstepping and sliding-mode methods (see details in [13]). However, physical implementation can be limited because of only partial or imprecise information is available from measurement of state and parameter values. Hence, in order to overcome this difficulty, estimation of state and identification of parameter values is needed. An alternative is use of adaptive observer-based schemes. An adaptive observer can be interpreted as a virtual (software) sensor for simultaneous estimation and identification. In this section, three different control techniques are shown which are based on an unique adaptive observer. In seek of completeness and simplicity in presentation, convergence and closed-loop stability proofs are sketched. More details can be found the original contributions [15, 16].

3.1 The control approaches

***State Feedback Linearization*:** This technique exploits differential geometry tools and has been one of the most popular in control practice since 80's. Then, firstly P-class chaotic systems is written in the following control affine form:

$$\begin{cases} \dot{w} &= f_l(w;\alpha) + g_l(w;\alpha)u_L \\ y_c &= h_l(w) \end{cases} \tag{3}$$

where $w \in \mathbb{R}^3$, $u_L \in \mathbb{R}$, $y_c \in \mathbb{R}$ are state vector, control input and output, respectively. Functions f_l and g_l are smooth vector fields. For a given continuous function $h_l : \mathbb{R}^3 \to \mathbb{R}$, Lie derivative is given by $L_{f_l}h_l(w) = \frac{\partial h_l}{\partial w} f_l(w;\alpha)$. Then, state feedback control

$$u_L(w;\alpha) = \frac{(-L_{f_l}^p h_l(w) + v_L)}{L_{g_l} L_{f_l}^{p-1} h_l(w)} \tag{4}$$

induces a linear behavior, where $v_L = (-k_1 y_c - k_2 y_c^{(1)} - ... - k_p y_c^{(p-1)})$. The constants k_i ($i = \{1, 2, ..., p\}$) are such that $s^p + k_p s^{p-1} + ... + k_2 s + k_1$ is a Hurwitz polynomial.

***Backstepping Control*:** This technique is based on solving a sequence of first-order systems succeeding is backward configuration. The method starts by writing the P-class system at the form:

$$\dot{\varpi} = f_\varpi(\varpi;\alpha) + g_\varpi(\varpi;\alpha)\varsigma \tag{5}$$

$$\dot{\varsigma} = f_\varsigma(\varpi,\varsigma;\alpha) + g_\varsigma(\varpi,\varsigma;\alpha)u_B \tag{6}$$

where $(\varpi, \varsigma) \in \mathbb{R}^2 \times \mathbb{R}$ is state vector and $u_B \in \mathbb{R}$ stands for control. Functions $f_\varpi : \mathbb{D} \to \mathbb{R}^2$ and $g_\varpi : \mathbb{D} \to \mathbb{R}^2$ are smooth in $\mathbb{D} \subset \mathbb{R}^2$ that contains $\varpi = 0$ and $f_\varpi(0;\alpha) = 0$.

System (5)-(6) is a cascade connection of two components. First component (5) with ς as the virtual control. Second component is the integrator (6). Assuming that (5) can be stabilized by a smooth state feedback control $\varsigma = \phi_B(\varpi)$, with $\phi_B(0) = 0$, the origin of $\dot{\varpi} = f_\varpi(\varpi;\alpha) + g_\varpi(\varpi;\alpha)\phi_B(\varpi)$ is asymptotically stable [13]. Thus, by means of the change of variables $z_B = \varsigma - \phi_B(\varpi)$, it follows that system

$$\left\{ \begin{array}{rcl} \dot{\varpi} & = & [f_\varpi(\varpi;\alpha) + g_\varpi(\varpi;\alpha)\phi_B(\varpi)] + g_\varpi(\varpi;\alpha) z_B \\ \dot{z}_B & = & u_B - \dot{\phi}_B \end{array} \right. \tag{7}$$

can be derived. Hence, by computing $\dot{\phi}_B$, and using the Lyapunov function $V_B(\varpi)$, the state feedback control law can be derived

$$u_B(\varpi,\varsigma;\alpha) = \frac{\partial \phi_B}{\partial \varpi}[f_\varpi(\varpi;\alpha) + g_\varpi(\varpi;\alpha)\varsigma] - \frac{\partial V_B}{\partial \varpi} g_\varpi(\varpi;\alpha) - k[\varsigma - \phi_B(\varpi)] \tag{8}$$

where k is a positive real constant.

***Sliding-mode Control*:** This technique has been used due to its robustness properties. An important feature is that control leads system trajectories at finite time towards a manifold and holds them on it. The manifold is constructed in terms of desired reference. Then, once the trajectories reach the sliding manifold, they tend to desired reference. P-class is written in the form to design controller

$$\dot{\eta} = f_\eta(\eta,\xi;\alpha) + \delta_\eta(\eta,\xi;\alpha) \tag{9}$$

$$\dot{\xi} = f_\xi(\eta,\xi;\alpha) + G_\xi(\eta,\xi;\alpha)[u_S + \delta_\xi(\eta,\xi,u_S;\alpha)] \tag{10}$$

where $\eta \in \mathbb{R}^2$, $\xi \in \mathbb{R}$, $u_S \in \mathbb{R}$ and δ_η and δ_ξ denote uncertainties. Let us consider the subsystem (9), where ξ is interpreted as a secondary control. By defining $\xi = \phi_S(\eta)$, where $\phi_S(\eta)$ is a smooth function satisfying $\phi_S(0) = 0$, the origin of (9) is asymptotically stable. Now, let us take $z_S = \xi - \phi_S(\eta)$. Control u entering into (10) leads $z_S(t)$ to zero, in finite time, holding it along time. Note dynamics of z_S is $\dot{z}_S = f_\xi(\eta,\xi;\alpha) + G_\xi(\eta,\xi;\alpha)[u_S + \delta_\xi(\eta,\xi,u;\alpha)] - \frac{\partial \phi_S}{\partial \eta}[f_\eta(\eta,\xi;\alpha) + \delta_\eta(\eta,\xi;\alpha)]$. Hence, the controller becomes

$$u_S(\eta,\xi;\alpha) = u_{eq}(\eta,\xi;\alpha) + G_\xi^{-1}(\eta,\xi;\alpha)v(\eta,\xi) \tag{11}$$

where $u_{eq}(\eta,\xi;\alpha) = G_\xi^{-1}(\eta,\xi;\alpha)[-f_\xi(\eta,\xi;\alpha) + \frac{\partial \phi}{\partial \eta} f_\eta(\eta,\xi;\alpha)]$ is chosen to cancel terms in $G_\xi(\eta,\xi;\alpha)$. $v(\eta,\xi)$ is determined by substituting $u_S(\cdot)$ into $\dot{z}_S$-equation to have

$$\dot{z}_S = v(\eta,\xi) + \Delta(\eta,\xi,v;\alpha) \tag{12}$$

Assuming Δ satisfies $\|\Delta(\eta,\xi,v;\alpha)\|_\infty \leq \rho_S(\eta,\xi;\alpha) + k\|v\|_\infty$, where $\rho_S(\eta,\xi;\alpha) \geq 0$ and $k \in [0,1)$ are known. Then, $v(\eta,\xi)$ is designed such that $z_S(t)$ is forced towards the manifold $z_S = 0$. In particular, $v(\eta,\xi)$ is chosen to be $v(\eta,\xi) = -\frac{\beta(\eta,\xi)}{1-k} sgn(z_S)$, where $\beta(\eta,\xi) \geq \rho_S(\eta,\xi;\alpha) + b_o$ for any constant $b_o > 0$, and $sgn(\cdot)$ is the signum function. Closed-loop stability of (12) can be ensured by taking the following Lyapunov function $V_S = \frac{1}{2}z_S^2$. As chattering is exhibited, as higher order sliding-mode can be designed to improve robustness and to diminish chattering.

3.2 Adaptive Observer

P-class system has only one unknown parameter and a single output $y_m \in \mathbb{R}$ available for feedback. Under its features, this P-class can be represented as a state affine system with unknown parameters as in [14, 15]:

$$\begin{cases} \dot{z} &= A(u,y_m)z + \varphi(u,y_m) + \Phi(u,y_m)\theta \\ y_m &= Cz \end{cases} \tag{13}$$

where entries of $A(u,y_m)$, $\varphi(u,y_m)$ and $\Phi(u,y_m)$ are continuous functions depending on u and y_m uniformly bounded and θ is a vector of unknown parameters. The following assumptions are introduced [16]:

Assumption 1 *There exists a bounded time-varying matrix $K(t)$ such that the following system $\dot{\Lambda}(t) = (A(t) - K(t)C(t))\Lambda(t)$ is exponentially stable.*

Assumption 2 *The solution $\Lambda(t)$ of $\dot{\Lambda}(t) = [A(t) - K(t)C(t)]\Lambda(t) + \Phi(t)$ is persistently exciting in the sense that exist α_1, β_1, T_1 such that*

$$\alpha_1 I \leq \int_t^{t+T_1} \Lambda(\tau)^T C^T \Sigma C(\tau)\Lambda(\tau)d\tau \leq \beta_1 I \tag{14}$$

for some bounded positive definite matrix Σ.

Assumption 3 *Control u is persistently exciting in the sense that there exist α_2, β_2, $T_2 > 0$ and $t_0 \geq 0$ such that:*

$$\alpha_2 I \leq \int_t^{t+T_2} \Psi_u(\tau,t)^T C^T \Sigma C(\tau)\Psi_u(\tau,t)d\tau \leq \beta_2 I \tag{15}$$

$\forall t \geq t_0$, where Ψ_u denotes the transition matrix for the system $\dot{z} = A(u,y_m)z$, $y_m = Cz$, and Σ some positive definite bounded matrix.

From (15) and (14) with $K = S^{-1}C^T$ and S as solution of $\dot{S} = -\rho S - A(u, y_m)^T S - SA(u, y_m) + C^T \Sigma C$, adaptive observer is given by

$$\left\{ \begin{array}{rcl} \dot{\hat{z}} & = & A(u, y_m)\hat{z} + \varphi(u, y_m) + \Phi(u, y_m)\hat{\theta} + \{\Lambda S_\theta^{-1}\Lambda^T C^T + S_z^{-1}C^T\}\Sigma(y_m - C\hat{z}) \\ \dot{\hat{\theta}} & = & S_\theta^{-1}\Lambda^T C^T \Sigma(y_m - C\hat{z}) \\ \dot{\Lambda} & = & \{A(u, y_m) - S_z^{-1}C^T C\}\Lambda + \Phi(u, y_m) \\ \dot{S}_z & = & -\rho_z S_z - A(u, y_m)^T S_z - S_z A(u, y_m) + C^T \Sigma C \\ \dot{S}_\theta & = & -\rho_\theta S_\theta + \Lambda^T C^T \Sigma C \Lambda \end{array} \right. \tag{16}$$

where $S_z(0) > 0$ and $S_\theta(0) > 0$, and ρ_z and ρ_θ are positive constants sufficiently large and Σ some bounded positive definite matrix. Moreover, it is remarkable that if Assumptions 2 and 3 are verified, they ensures that the matrices S_z and S_θ are invertible and symmetric positive definite. Now, we can establish the following result.

Lemma 1 *Let us consider system (13). Suppose 1, 2 and 3 hold. Then, system (16) is an adaptive observer for system (13). That is, estimation error vector ($e_z := \hat{z} - z$, $\epsilon_\theta := \hat{\theta} - \theta$) converges exponentially to zero with a rate given by $\rho = \min(\rho_z, \rho_\theta)$.*

Sketch of the Proof. Let $e_z := \hat{z} - z$ and $\epsilon_\theta := \hat{\theta} - \theta$ be the convergence errors for states and parameter, respectively. Defining $\epsilon_z = e_z - \Lambda\epsilon_\theta$, it follows that

$$\dot{\epsilon}_z = \{A(u, y_m) - \Lambda S_\theta^{-1}\Lambda^T C^T \Sigma C - S_z^{-1}C^T \Sigma C\}e_z + \Phi(u, y_m)\epsilon_\theta - \dot{\Lambda}\epsilon_\theta - \Lambda\dot{\epsilon}_\theta$$

Replacing suitable expressions in above equation, we get:

$$\left\{ \begin{array}{rcl} \dot{\epsilon}_z & = & \{A(u, y_m) - S_x^{-1}C^T \Sigma C\}\epsilon_z \\ \dot{\epsilon}_\theta & = & -S_\theta^{-1}\Lambda^T C^T \Sigma C(\epsilon_z + \Lambda\epsilon_\theta) \end{array} \right.$$

We consider $V(\epsilon_z, \epsilon_\theta) = \epsilon_z^T S_z \epsilon_z + \epsilon_\theta^T S_\theta \epsilon_\theta$ as a Lyapunov function to prove observer convergence. Then, taking $\dot{V}$ and replacing appropriated expressions, we obtain $\dot{V}(\epsilon_z, \epsilon_\theta) \leq -\rho_z \epsilon_z^T S_z \epsilon_z - \rho_\theta \epsilon_\theta^T S_\theta \epsilon_\theta$. Taking $\rho = \min(\rho_z, \rho_\theta)$, we have $\dot{V}(\epsilon_z, \epsilon_\theta) \leq -\rho V(\epsilon_z, \epsilon_\theta)$. Finally, ϵ_z and ϵ_θ converge to zero exponentially with a rate given by ρ. This ends the proof.

Note that the chaotic system (1) can be represented in the following general form

$$\dot{\mathbf{x}} = f(\mathbf{x}; \alpha), \quad y_m = C\mathbf{x} \tag{19}$$

where $\mathbf{x}$ is state vector, y_m is measured output and constant α is a real parameter whose exact value in unknown. By means of a change of coordinates ($z = T(\mathbf{x})$), (19) can be transformed into a state-affine system (13), for which is possible to design adaptive observer (16).

3.3 Closed-loop stability

Now, closed-loop system is analyzed with the three first controllers (4), (8), and (11) based on adaptive observer (16). From system (13), we have that by extending the state vector by parameters vector θ, into $Z := (z \quad \theta)^T$, state affine structure is preserved as follows:

$$\begin{cases} \dot{Z} &= F(\vartheta)Z + G(\vartheta) \\ y_m &= HZ \end{cases} \tag{20}$$

where $\vartheta := (u \quad y_m)$, $H = (C \quad 0)$, $F(\vartheta) = \begin{pmatrix} A(\vartheta) & \Phi(\vartheta) \\ 0 & 0 \end{pmatrix}$, and $G(\vartheta) = \begin{pmatrix} \varphi(\vartheta) \\ 0 \end{pmatrix}$.

Thus, extended system is given by

$$\begin{cases} \dot{Z} &= F(\vartheta(\hat{Z}))Z + G(\vartheta(\hat{Z})) \\ \dot{\hat{Z}} &= F(\vartheta(\hat{Z}))\hat{Z} + G(\vartheta(\hat{Z})) + S^{-1}H^T(y_m - H\hat{Z}) \\ \dot{S} &= -\rho S - F^T(\vartheta(\hat{Z}))S - SF(\vartheta(\hat{Z})) + H^T H \end{cases} \tag{21}$$

where $S = \begin{pmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{pmatrix}$, and S_z, S_θ, Λ of (16) are related to solution S through:

$$\begin{cases} S_z &= S_1 \\ S_\theta &= S_3 - S_2^T S_1^{-1} S_2 \\ \Lambda &= -S_1^{-1} S_2 \end{cases} \tag{22}$$

Let us define $e := \hat{Z} - Z$ be the estimation error. Then, the dynamics of resulting observer-based controller can be rewritten as:

$$\begin{cases} \dot{e} &= \{F(\vartheta(\hat{Z})) - S^{-1}H^T H\}e \\ \dot{Z} &= F(\vartheta(\hat{Z}))Z + G(\vartheta(\hat{Z})) \\ \dot{S} &= -\theta S - F^T(\vartheta(\hat{Z}))S - SF(\vartheta(\hat{Z})) + H^T H \end{cases} \tag{23}$$

where $u(\hat{Z})$ is the controller given by (4), (8), or (11) for each case. Next lemma is stated to prove closed-loop stability:

Lemma 2 *[16] Assume that ϑ is regularly persistent for (20) and let be*

$$\dot{S}(t) = -\theta S(t) - F^T(\vartheta(\hat{Z}))S(t) - S(t)F(\vartheta(\hat{Z})) + H^T H$$

a Lyapunov differential equation, with $S(0) > 0$. Then, $\exists\ \theta_0 > 0$ such that for any symmetric positive definite matrix $S(0); \forall \theta \geq \theta_0$

$$\exists\ \bar{\alpha} > 0, \bar{\beta} > 0, t_0 > 0 : \forall t > t_0$$

$$\bar{\alpha} I \leq S(t) \leq \bar{\beta} I$$

Then, the following result can be established:

Theorem 1 *Under assumptions that nominal controller is globally asymptotically stable and that state Z in (21) remains for all $t > t_0 > 0$ in a compact set Ω (containing equilibrium point of nominal controller) $\forall\ Z(0) \in \Omega$, extended system (21) is globally asymptotically stable on $\Omega \times \mathbb{R}^n \times S_n^+$ (i.e., $\forall\ Z(0) \in \Omega, \forall\ \hat{Z}(0) \in \mathbb{R}^n; \forall\ S(0) > 0$).*

Proof. Since observer (16) is such that estimation error goes to zero, it is bounded and matrix S is solution of the Lyapunov differential equation in (21), is bounded from above and from below in set of positive definite matrices, see Lemma 2. Hence, whole state $e = (\hat{Z} - Z, \hat{Z}, S)$ of (21) remains in a compact set along any trajectory.

Let $\Lambda = \{(e(t), \hat{Z}(t), S(t)), t \geq 0\}$ be a semitrajectory of observer-based controller given by (21). This semitrajectory, lying in a compact set, has a nonempty ω-limit set (i.e., ω-limit set of a trajectory is set of its accumulation points). Let $[\bar{e}, \bar{Z}, \bar{S}]$ be an element of ω-limit set of Λ. It is clear that when $e \to 0$ implies that $\bar{e} = 0$. Let $\{(0, \hat{Z}(t), S(t)), t \geq 0\}$ be a semitrajectory starting at time $t = 0$ from $[0, \bar{Z}, \bar{S}]$. Since the ω-limit set is positively invariant, it follows that the semitrajectory $\{(0, \hat{Z}(t), S(t)), t \geq 0\}$ belongs to ω-limit set of Λ. The estimation error is here equal to zero for this semitrajectory, and using our closed-loop stability assumption, $\hat{Z}$ is globally asymptotically stable, i.e., $\hat{Z}(t) \to Z^* = \psi(Z^*)$. So, there are points at which $e = 0$ and $\hat{Z} = Z^*$ in ω-limit set of Λ, since it is a closed set. Letting $[0, Z^*, \bar{S}(t)]$ be an element of ω-limit set of Λ and following same reasoning: let $\{(0, Z^*, S(t)), t \geq 0\}$ be a semitrajectory starting at $t = 0$ from $[0, Z^*, \bar{S}]$. This semitrajectory belongs to ω-limit set of Λ. The dynamics of $S(t)$ are given by Lyapunov differential equation and using the observability of constant linear system $(F^* + G^*, H)$, that $S(t)$ tends to S^*, unique positive definite solution of Lyapunov algebraic equation. So, $[0, Z^*, S^*]$ belongs to ω-limit set of Λ. It follows that, under the assumption of (local) asymptotic stability of (21), Λ enters in a finite time into the basin of attraction of $[0, Z^*, S^*]$. Hence (21) is globally asymptotically stable on $\Omega \times \mathbb{R}^n \times S_n^+$.

4 Robust Control with Low-order Parametrization

Above controllers are compared with other adaptive scheme previously reported [5]. This last adaptive scheme compensates uncertainties in parameters and unmeasured states with a low order equation. Let us re-write P-class system in the form (1) in the canonical form: $\dot{\chi}_i = \chi_{i+1}$ with $1 \leq i \leq p-1$, $\dot{\chi}_p = f_p(\chi, \nu; \alpha) + \Gamma(\chi, \nu; \alpha) u_A$ and $\dot{\nu} = \zeta(\chi, \nu)$. Then, by exploiting Lie algebra of vectors fields, P-class system can be stabilized as p is a integer constant such that $\Gamma(\chi, \nu; \alpha) \neq 0$ at any point belonging to domain (see details in [5]). Functions $f_p(\chi, \nu; \alpha)$ and $\Gamma(\chi, \nu; \alpha)$ are assumed uncertain and unavailable for feedback. Thus, the $\gamma(\chi, \nu; \alpha) = \Gamma(\chi, \nu; \alpha) - \Gamma_{nom}(\chi)$ and $\Theta(\chi, \nu, u_A; \alpha) = f_p(\chi, \nu; \alpha) + \gamma(\chi, \nu; \alpha) u_A$ can be defined. After algebraic manipulations, we have:

$$\left\{ \begin{array}{rcll} \dot{\chi}_i &=& \chi_{i+1}, & 1 \leq i \leq p-1 \\ \dot{\chi}_p &=& \Theta(\chi, \nu, u_A; \alpha) + \Gamma_{nom}(\chi) u_A & \\ \dot{\nu} &=& \zeta(\chi, \nu) & \end{array} \right. \tag{24}$$

where $\Theta(\chi, \nu, u_A; \alpha)$ is a continuous function that lumps the uncertain terms. Lumping function Θ is seen as an augmented state by defining $\Theta \equiv \Theta(x, \nu, u_A; \alpha)$. So, system (24) can be rewritten in the augmented form

$$\left\{ \begin{array}{rcll} \dot{\chi}_i &=& \chi_{i+1}, & 1 \leq i \leq p-1 \\ \dot{\chi}_p &=& \Theta + \Gamma_{nom}(\chi) u_A & \\ \dot{\Theta} &=& \Xi(x, \nu, \Theta, u_A; \alpha) & \\ \dot{\nu} &=& \zeta(\chi, \nu) & \end{array} \right. \tag{25}$$

where $\Xi(x, \nu, \Theta, u_A; \alpha) = \sum_{i=1}^{p-1} \chi_{i+1} \partial_i \Theta(x, \nu, u_A; \alpha) + [\Theta + \Gamma_{nom}(\chi) u_A] \partial_p \Theta(x, \nu, u_A; \alpha) + \gamma(\chi, \nu; \alpha) \dot{u}_A + \partial_t \gamma(\chi, \nu; \alpha) u_A + \partial_\nu \Theta(x, \nu, u_A; \alpha) \zeta(\chi, \nu)$.

Controller $u_A(\chi, \Theta) = \frac{1}{\Gamma_{nom}(\chi)}(-\Theta - K^T \chi)$ can be used to stabilize (25), where $K \in \mathbb{R}^p$ is chosen in such way that $P_K(s) = s^p + k_p s^{p-1} + ... + k_2 s + k_1 = 0$ is Hurwitz. Now, problem of estimating (χ, Θ) can be addressed by using a high-gain observer for (25). Thus, dynamics of χ and Θ is reconstructed from measurements of output $y_m = \chi_1$ by

$$\left\{ \begin{array}{rcll} \dot{\hat{\chi}}_i &=& \hat{\chi}_{i+1} + L^i \kappa_i (\chi_1 - \hat{\chi}_1), & 1 \leq i \leq p-1 \\ \dot{\hat{\chi}}_p &=& \hat{\Theta} + \Gamma_{nom}(\hat{\chi}) u_A + L^p \kappa_p (\chi_1 - \hat{\chi}_1) & \\ \dot{\hat{\Theta}} &=& L^{p+1} \kappa_{p+1} (\chi_1 - \hat{\chi}_1) & \end{array} \right. \tag{26}$$

where κ_j, $j = 1, 2, ..., p+1$, are such that $P_\kappa(s) = s^{p+1} + \kappa_1 s^p + ... + \kappa_p s + \kappa_{p+1} = 0$ is also Hurwitz. Parameter $L > 0$ stands for estimation rate of uncertainties, being unique tuning parameter. Finally, control becomes

$$u_A(\hat{\chi}, \hat{\Theta}) = \frac{1}{\Gamma_{nom}(\hat{\chi})}(-\hat{\Theta} - K^T \hat{\chi}) \tag{27}$$

5 On experimental implementation

Implementation is realized to compare controllers performance. Let us consider only one of chaotic systems in collection (Σ_2). Only $y_m = x_2$ is measured and parameter α is uncertain, the problem is to lead output $y_c = x_1$ (note $y_m \neq y_c$) when u is acting on right-side of (Σ_2) as follows

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -\alpha x_3 - x_1 + x_1 x_2 + u \end{cases} \tag{28}$$

5.1 Implemented controllers

State feedback linearizating control:

For (28), we have that $f_l(w;\alpha) = [x_2, x_3, -\alpha x_3 - x_1 + x_1 x_2]^T$ and $g_l(w;\alpha) = [0, 0, 1]^T$. Then, designed controller is given by

$$u_L(w;\alpha) = \alpha x_3 + x_1 - x_1 x_2 - k_1 x_1 - k_2 x_2 - k_3 x_3 \tag{29}$$

Backstepping control:

Let us write the subsystem

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \end{cases} \tag{30}$$

which is in form (5), where $\bar{\varpi} = x_1$, $\bar{\varsigma} = x_2$ and $\bar{u}_B = x_3$. By proposing $x_2 = \bar{\phi}_B(\bar{\varpi}) = -\mu x_1$, where μ is a positive real constant, and $\bar{V}_B(\bar{\varpi}) = \frac{1}{2}x_1^2$ is a Lyapunov function, then the controller (8) for (30) is given by

$$\bar{u}_B(\bar{\varpi}, \bar{\varsigma}) = x_3 = -\mu x_2 - x_1 - \bar{k}(x_2 + \mu x_1) = \phi_B(x_1, x_2) \tag{31}$$

with $\bar{k} > 0$. Now, let us consider the complete system (28) re-written as

$$\left\{ \begin{array}{rcl} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} & = & \begin{pmatrix} x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x_3 \\ \dot{x}_3 & = & (-\alpha x_3 - x_1 + x_1 x_2) + u_B \end{array} \right. \tag{32}$$

which is in form (5)-(6), where $\varpi = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T$ and $\varsigma = x_3$. Taking (31), $k > 0$, and $V_B(\varpi) = \frac{1}{2}x_1^2 + \frac{1}{2}[x_2 + \mu x_1]^2$ as Lyapunov function, then controller (8) for system (28) is given by:

$$\left\{ \begin{array}{rcl} u_B(\varpi, \varsigma; \alpha) & = & u_{Ba} - (1 + \bar{k}\mu)x_2 - (\mu + \bar{k})x_3 - (x_2 + \mu x_1) \\ & & -k[x_3 + (1 + \bar{k}\mu)x_1 + (\mu + \bar{k})x_2] \\ u_{Ba}(\varpi, \varsigma; \alpha) & = & \alpha x_3 + x_1 - x_1 x_2 \end{array} \right. \tag{33}$$

Sliding-mode control:

By defining $\eta = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T$ and $\xi = x_3$,(28) can be written as

$$\dot{x}_1 = x_2 + \delta_{\eta 1} \tag{34}$$

$$\dot{x}_2 = x_3 + \delta_{\eta 2} \tag{35}$$

$$\dot{x}_3 = [-\alpha_0 x_3 - x_1 + x_1 x_2] + [u_S + \delta_\xi] \tag{36}$$

where $\delta_{\eta 1}$, $\delta_{\eta 2}$ and δ_ξ are the uncertainties. Sliding surface is given by $z_S = x_3 + \sigma_1 x_1 + \sigma_2 x_2$, and from (36) and controller takes the form

$$\left\{ \begin{array}{rcl} u_S(\eta, \xi) & = & u_{eq}(\eta, \xi) - \frac{\beta(\eta,\xi)}{(1-k)} sgn(x_3 + \sigma_1 x_1 + \sigma_2 x_2) \\ u_{eq}(\eta, \xi) & = & [\alpha_0 x_3 + x_1 - x_1 x_2 - \sigma_1 x_2 - \sigma_2 x_3] \\ \beta(\eta, \xi) & = & k_1|x_1||x_2| + k_2|x_1| + k_3|x_2| + k_4|x_3| + b_o \end{array} \right. \tag{37}$$

5.2 State estimation and parameter identification

Control implementation requires state estimation and parameter identification provided by adaptive observer given in Section 4. Then, system structure is transformed into

$$\left\{ \begin{array}{rcl} \dot{z}_1 & = & -\alpha z_1 + z_2 \\ \dot{z}_2 & = & -z_3 + z_1 z_3 + u \\ \dot{z}_3 & = & z_1 \end{array} \right. \tag{38}$$

By defining $\theta = \alpha$ and $y_m = z_1$, adaptive observer (16) is derived. Then, states and parameter are replaced in controllers (29), (33), and (37) becomes

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & z_1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} -z_1 \\ 0 \\ 0 \end{pmatrix} \alpha + \begin{pmatrix} 0 \\ u \\ z_1 \end{pmatrix}$$

5.3 Robust Control with Low-order Parametrization

By defining $\chi = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$, $\Theta = -\alpha x_3 - x_1 + x_1 x_2$ and $\Gamma_{nom}(\chi) = 1$; system (28) is written as (25) to have:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= \Theta + u_A \\ \dot{\Theta} &= \Xi(x, \Theta, u_A; \alpha) \end{cases} \tag{40}$$

Hence, (26) and (27) becomes

$$\begin{cases} \dot{\hat{x}}_1 &= \hat{x}_2 + L\kappa_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + L^2\kappa_2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 &= \hat{\Theta} + u_A + L^3\kappa_3(x_1 - \hat{x}_1) \\ \dot{\hat{\Theta}} &= L^4\kappa_4(x_1 - \hat{x}_1) \end{cases} \tag{41}$$

$$u_A(\hat{\chi}, \hat{\Theta}) = -\hat{\Theta} - k_1\hat{x}_1 - k_2\hat{x}_2 - k_3\hat{x}_3 \tag{42}$$

where L, κ_j $(j = 1, 2, 3, 4)$ are taken as (26). and k_i $(i = 1, 2, 3)$ are as in (27).

5.4 Performance index

Since control objective is to suppress chaotic behavior, control performance can be evaluated via a performance index. Next, three indexes are considered:

- The first one is a measure of stabilization error, equivalent to an index of chaos suppression, during the interval $[t_0, t_f]$. This index is defined by $J_s = \frac{1}{t_f - t_0}\int_{t_0}^{t_f}\{x(t)^T Q(t) x(t)\}dt$, where $Q(t)$ is a positive semi-definite symmetric matrix for all $t \in [t_0, t_f]$. $Q(t) = I(1 - e^{-\lambda(t-t_0)})^q$ was chosen to assign major weight to steady-state error, where I is the identity matrix, $\lambda > 0$, and q is a positive integer.

- In second index, control effort is also measured by its overshoot; which is defined by infinity norm $u_{max} =^{\max}_{[t_0,t_f]} \text{abs}(u(t))$.

- Finally, the last index is used to measure average control effort at implementation interval $[t_0, t_f]$. This index is defined by $J_{ue} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \{u(t)^T Q(t) u(t)\} dt$.

5.5 Physical implementation and performance comparison

System (28) was electronically realized by means of operational amplifiers TL084CN, an analog multiplier AD633JN and passive components. A DSpace 1104 acquisition data board was used to measure control action, capture system state values along implementation time and estimate states and parameter values from adaptive observer. Figures 1 and 2 show, respectively, a photo of experimental setup and a detail of oscilloscope depicting time series during experiments. Schematic of used circuit is in Figure 3.

In this way, control schemes were implemented selecting the following parameters:

- *State feedback linearization* (29): $k_1 = 27$, $k_2 = 27$ and $k_3 = 9$.

- *Backstepping* (33): $\mu = 1$, $k = 1$ and $\bar{k} = 1$.

- *Sliding-mode* (37): $\sigma_1 = 1$, $\sigma_2 = 1$, $k = 0.04$, $k_1 = 0.34$, $k_2 = 0.1$, $k_3 = 0.08$, $k_4 = 0.3224$, $b_o = 0.01$ and $\alpha_0 = 2.02$.

- *Adaptive Observer* (16): $\rho_z = 50$, $\rho_\theta = 2$, $S_z(0) = I$, $S_\theta(0) = I$ and $\Lambda(0) = [10, 10, 10]^T$.

- *Robust control with low-order parametrization* (41)-(42): $k_1 = 27$, $k_2 = 27$, $k_3 = 9$, $L = 10$, $\kappa_1 = 4$, $\kappa_2 = 6$, $\kappa_3 = 4$ and $\kappa_4 = 1$.

Experimental results are shown in the Figures 4, 5, 6, and 7. Now, a comparative study is presented for the four controllers. Performance indexes are evaluated. Table 1 shows indexes values from evaluation of each controller.

Table 1: Performance indexes from experimental implementation.

Control strategy	J_s	u_{max}	J_{ue}
Linearization state-feedback	0.026381	43.7965	0.0002980
Backstepping	0.030910	4.0440	0.0000097
Sliding-mode	0.199010	3.9620	7.771480
Adaptive low-parametrized	0.818650	2.9216	0.601240

From Table 1, feedback linearization method shows a small stabilization error, i.e. a small J_s performance index value. Figure 4 shows the fastest response. However, the control effort shows largest overshoot. On the other hand, robust control with low parametrization shows lowest overshoot but largest average control effort and a slow response. Furthermore, lowest average control effort was found for backstepping controller and showed fast response and small overshoot. Finally, the sliding-mode methodology allows to have a strategy more robust. However, this scheme demands important average control effort. Furthermore, sliding-mode control shows chattering effects.

6 Conclusions

In this paper, a comparative study of control performance has been shown in regard to chaos suppression. Control strategies, based on an adaptive observer, have been presented to evaluate them: feedback linearization, backstepping and sliding-mode. Furthermore, the convergence of the adaptive observer has been shown, where sufficient conditions have been given. Then, a stability analysis of the closed loop system has been presented. Additionally, a robust control with low-order parametrization has been also considered in this comparative study.

The comparative study of these schemes has been done considering three performance indexes, which have been taken into account to measure stabilization error, control effort, and average control effort. Experimental results were measured to evaluate the performance of each scheme. As a summary, state feedback with adaptive observer yields the lowest stabilization error but largest overshoot. The lowest average control effort was

	Lorenz system	Chen System	Lü system
Equations:	$\dot{x}_1 = a(x_2 - x_1)$ $\dot{x}_2 = dx_1 - x_2 - x_1x_3$ $\dot{x}_3 = -bx_3 + x_1x_2$	$\dot{x}_1 = a(x_2 - x_1)$ $\dot{x}_2 = (c-a)x_1 + cx_2 - x_1x_3$ $\dot{x}_3 = -bx_3 + x_1x_2$	$\dot{x}_1 = a(x_2 - x_1)$ $\dot{x}_2 = cx_2 - x_1x_3$ $\dot{x}_3 = -bx_3 + x_1x_2$
Parameters:	$(a,b,c,d) = (10, \frac{8}{3}, -1, 28)$	$(a,b,c,d) = (35, 3, 28, -7)$	$(a,b,c,d) = (36, 3, 20, 0)$

Table 2: Lorenz-type systems

obtained by backstepping strategy. Lowest overshoot was exhibited by robust control with low-order parametrization, which had the largest stabilization error. It follows that backstepping method showed best performance.

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A Study cases

In this section we introduce a chaotic system which contains the Lorenz, Chen and Lü systems. Let be a Lorenz-type system given by

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ d & c & 0 \\ 0 & 0 & -b \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix} \tag{43}$$

where a, b, c and d are constants.In Table A, we list mathematical models of these three systems and corresponding parameter values.

Now, P-class contains the well known above systems. In this case, Lorenz-type system is chosen as an example to shown that after a change of coordinates it can be transformed into jerk equation (1). Let be the Lorenz system represented by

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ d & 1 & 0 \\ 0 & 0 & -b \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix} \tag{44}$$

Then, by defining output $y = x_1$, we derive a diffeomorphism $\Phi : \Re^3 \to \Re^3$ given by

$$\Phi(x) = \begin{bmatrix} x_1 \\ a(x_2 - x_1) \\ a(dx_1 - cx_2 - x_1 x_3 - a(x_2 - x_1)) \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \bar{z}_3 \end{bmatrix} = z$$

such that control affine system can be transformed into normal form:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= \alpha(z) + \beta(z)u \end{cases} \tag{45}$$

We can see that matrix

$$\frac{\partial \Phi(x)}{\partial x} = \begin{pmatrix} 1 & 0 & 0 \\ -a & a & 0 \\ a(d - a - x_3) & -a(c + a) & -ax_1 \end{pmatrix}$$

is singular only at $\mathcal{S} = x|(x_1 = 0, x_2, x_3)$. Note that affine system cannot be stabilized at origin because of it has no well-posed relative degree $\rho = 3$ for any $x \in \mathcal{S}$.

Other system that can be transformed into jerk equation (1) is Rössler, which is in affine form:

$$\begin{cases} \dot{x}_1 = & -x_2 - x_3 \\ \dot{x}_2 = & x_1 + ax_2 \\ \dot{x}_3 = & x_3(x_1 - b) + a \end{cases} \tag{46}$$

or

$$\dot{x} = \begin{pmatrix} -x_2 - x_3 \\ x_1 + ax_2 \\ x_3(x_1 - b) + a \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ g_3 \end{pmatrix} u \tag{47}$$

where a and b are nonzero constants. Then, by defining $y = x_2$, we can also derive a diffeomorphism $\Phi : \Re^3 \to \Re^3$ given by

$$\Phi(x) = \begin{bmatrix} x_2 \\ x_1 + ax_2 \\ (a^2-1)x_2 - x_3 + ax_1 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \bar{z}_3 \end{bmatrix} = z$$

such that affine Rössler system under output $y = x_2$ can be transformed into normal form:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= \alpha(z) + \beta(z)u \end{cases} \tag{48}$$

where

$$\alpha(z) = -a(2z_1 - z_3 - az_2) + (a^2-1)(z_2 + 2az_1) - (z_1 - z_3 - az_2)(z_2 + az_1 - b) + a$$

$$\beta(z) = -g_3$$

and

$$x = \Phi^{-1}(z) = \begin{bmatrix} z_2 + az_1 \\ z_1 \\ z_1 - z_3 + az_2 \end{bmatrix}$$

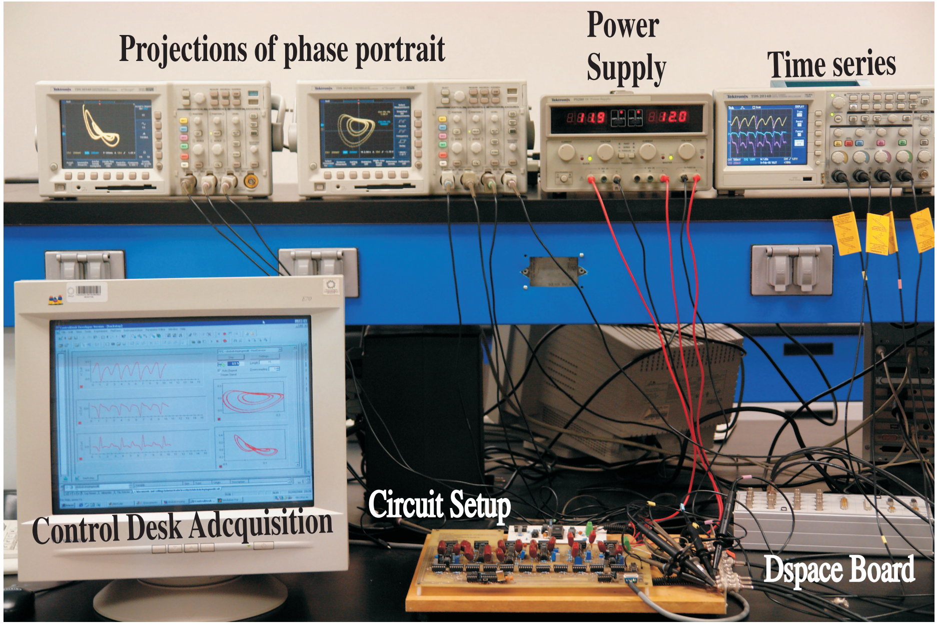

Figure 1: Photo of experimental setup.

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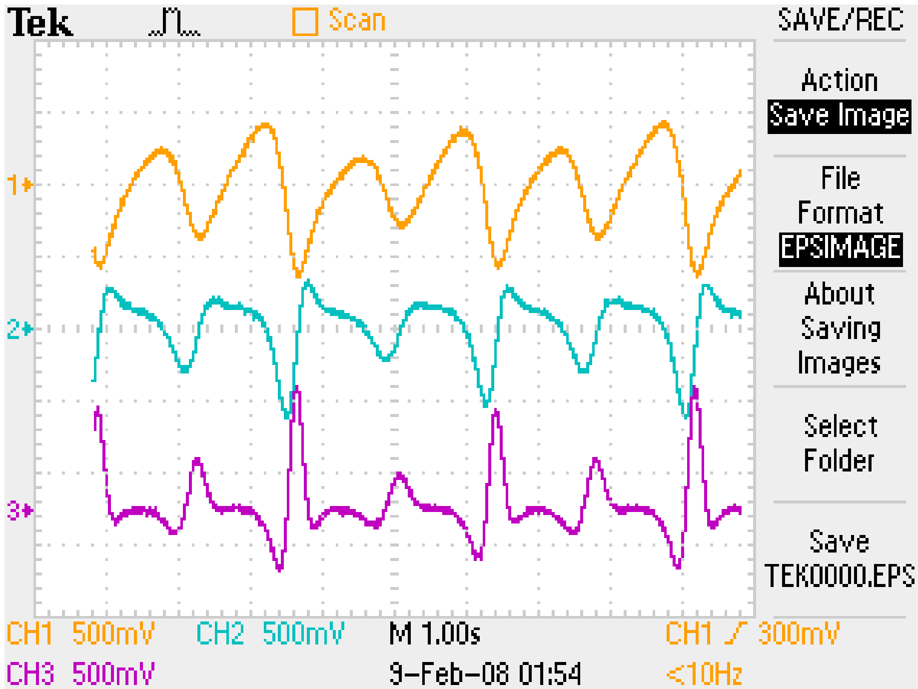

Figure 2: Detail of time series from photo in Figure 1.

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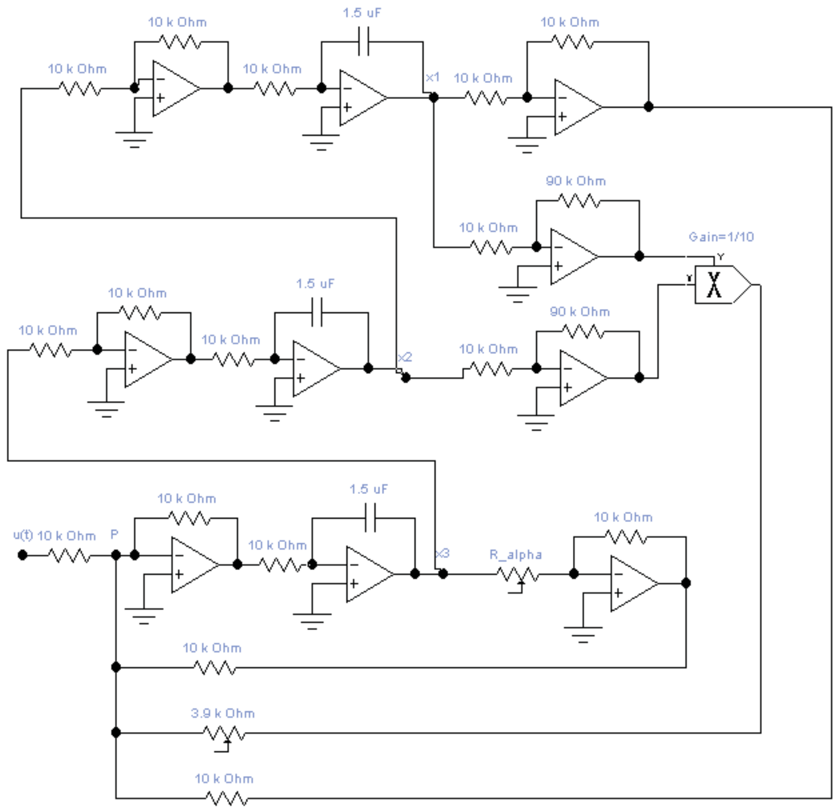

Figure 3: Schematic diagram for physical realization of the system (28).

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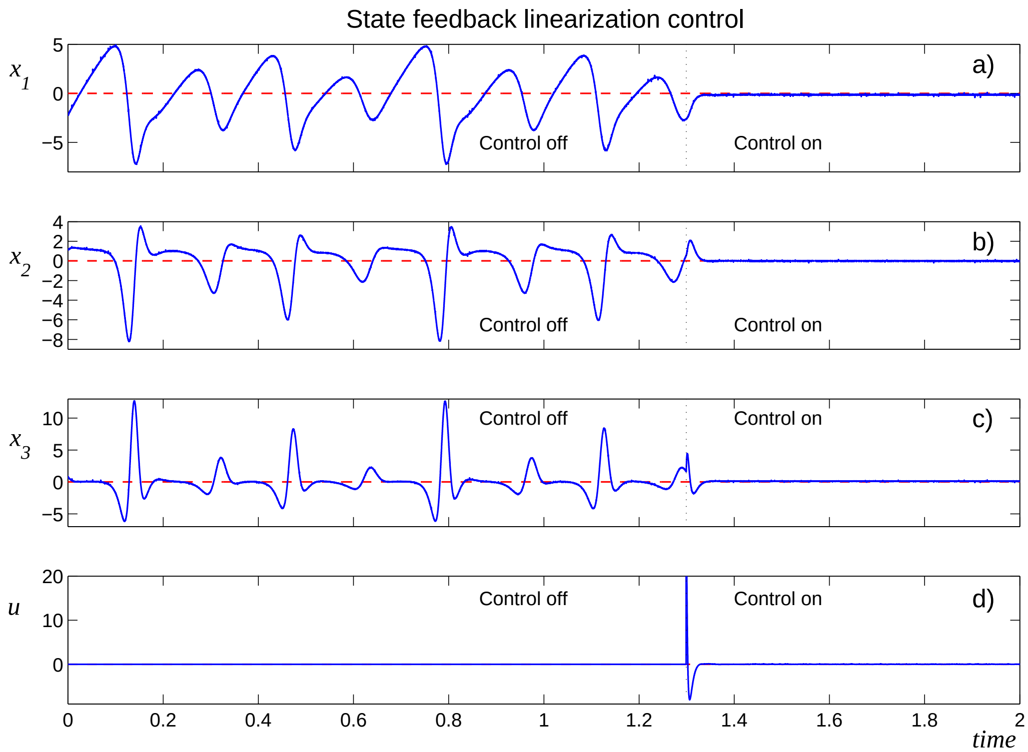

Figure 4: Responses with state feedback linearization control u_L.

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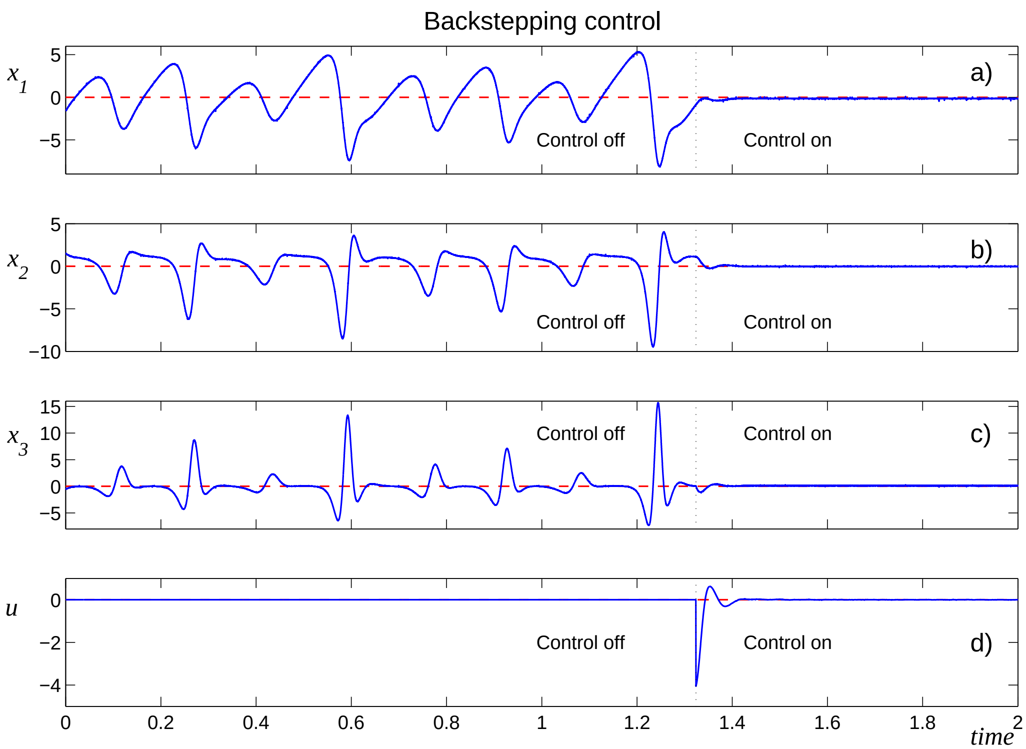

Figure 5: Responses with backsttepping control u_B.

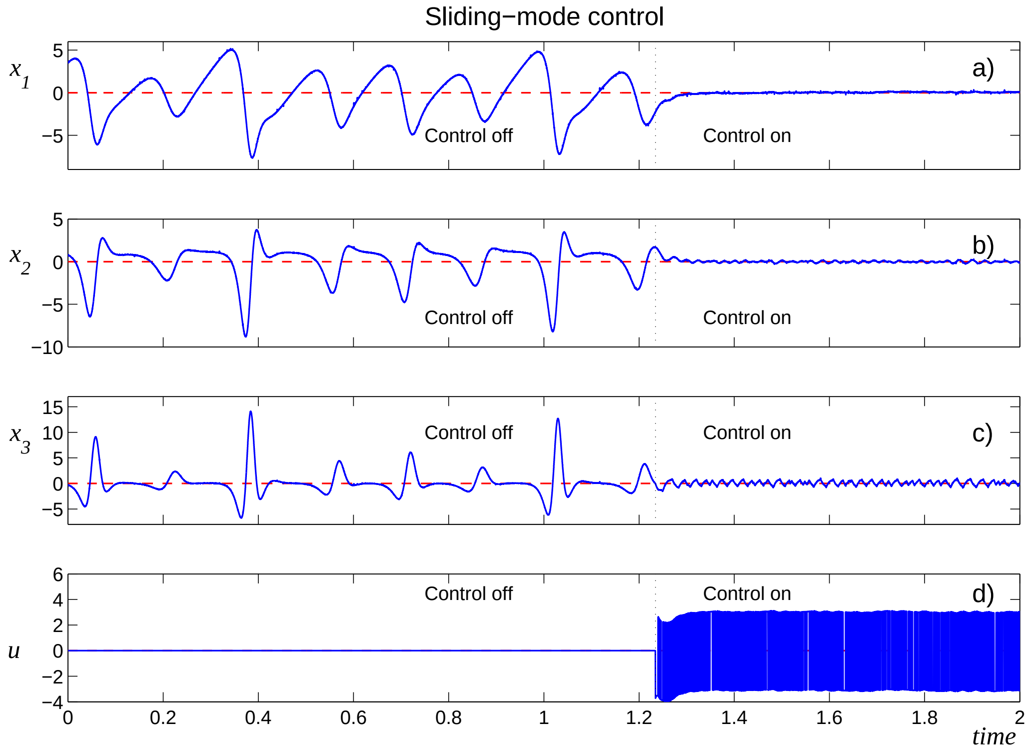

Figure 6: Responses with sliding-mode control u_S.

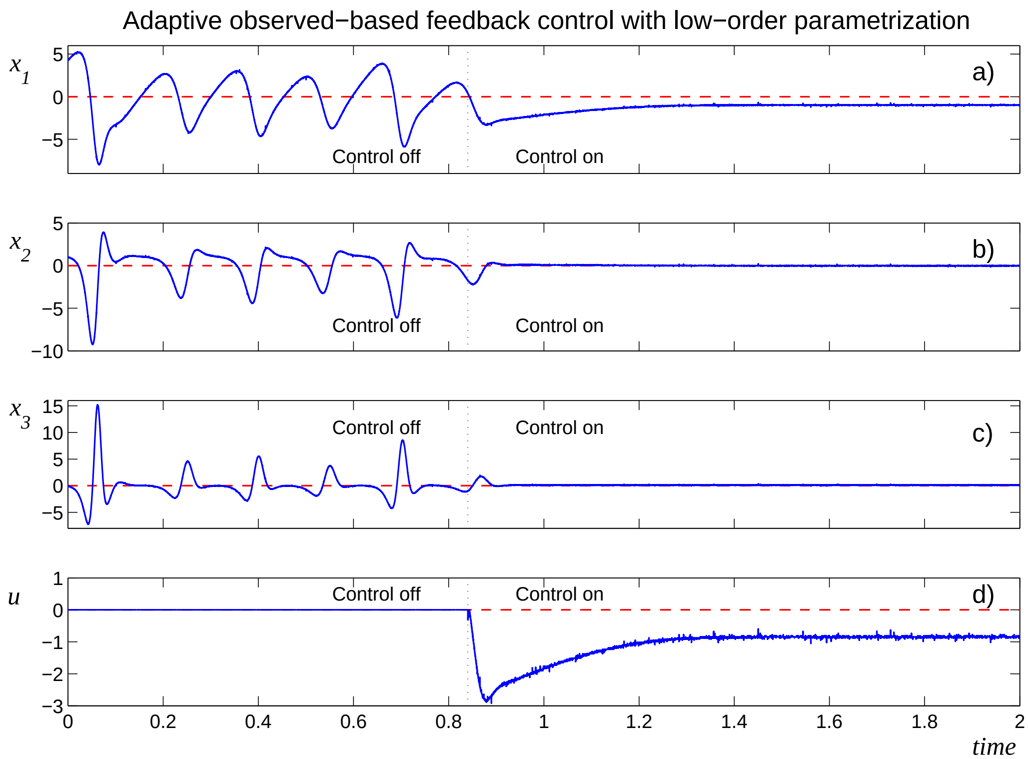

Figure 7: Responses with adaptive observed-based feedback control with low-order parametrization u_A.