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GLOBAL TRAJECTORY TRACKING THROUGH OUTPUT FEEDBACK FOR ROBOT MANIPULATORS WITH BOUNDED INPUTS

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ABSTRACT

In this work, a globally stabilizing output feedback scheme for the trajectory tracking of robot manipulators with bounded inputs is proposed. It achieves the motion control objective avoiding input saturation and excluding velocity measurements. Moreover, it is not defined using a specific sigmoidal function, but any one on a set of *saturation* functions. Consequently, the proposed scheme actually constitutes a family of globally stabilizing output feedback bounded controllers. Furthermore, the bound of such saturation functions is explicitly considered in their definition. Hence, the control gains are not tied to satisfy any *saturation-avoidance* inequality and may consequently take any positive value, which may be considered beneficial for performance adjustment/improvement purposes. Further, a class of *desired trajectories* that may be globally tracked avoiding input saturation and excluding velocity measurements is completely characterized. Global asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory is proved through a strict Lyapunov function. The efficiency of the proposed scheme is corroborated through experimental results.

Key Words: Global tracking, bounded inputs, output feedback, robot control, saturation functions.

I. INTRODUCTION

Two classical controllers for the trajectory tracking of n-degree-of-freedom (n-DOF) robot manipulators are the well-known *Computed-Torque* algorithm (see for instance [1, Chap. 10]) and the so-called *PD+* scheme [2]. The former results from the application of the *exact linearization via feedback* design methodology [3] to the manipulator dynamic model. Through this technique, exact compensation of the robot dynamics is carried out to impose a linear structure to the closed-loop system, expressed in terms of the *error variable* —defined as the position that the manipulator keeps with respect to the current coordinate vector of the desired trajectory on the configuration space—, with globally asymptotically stable trivial solution. The latter considers a continuous calculation of a special form of the robot dynamics, where the current position vector is considered at every of its terms (gravity, inertial, and centrifugal and Coriolis calculated force vectors), the desired acceleration vector is involved in the computed inertial force vector, and both the current and desired velocity vectors are considered in the Coriolis and centrifugal calculated force vector. This gives rise to a strategic closed loop form wherefrom it is clear that the desired trajectory is a solution of the closed-loop system. But such terms do not guarantee, by themselves, the (global) stabilization

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towards the desired trajectory. This is achieved through the additional consideration of position-error (P) and velocity-error (D) linear correction terms.

A common characteristic of both conventional controllers is that they are based on ideal assumptions. For instance, inputs capable to furnish any force or torque value, the availability of accurate measurements of all the system states, and the exact knowledge of the dynamic parameters are supposed. However, it is well-known that such assumptions are not compatible with real-life implementations. Input capabilities and the availability on the system data are generally limited. Moreover, the fact that such limitations are not considered by the control scheme may lead to unexpected or undesirable effects on the closed-loop performance —like input saturation and those related to such a nonlinear phenomenon [4], noisy responses and/or deteriorated performance [5], and inaccurate tracking [6]— which sometimes give rise to serious or dramatic consequences on the system responses [5], [7, §5.2], [8, §15.4].

In a bounded input context, there are several works proposing motion control schemes under the consideration of additional constraints (missing system information). For instance, an output feedback bounded dynamical extension of the PD+ algorithm is proposed in [9]. First of all, the current velocity vector is replaced by the desired velocity trajectory in the computed Coriolis and centrifugal force vector. Hence, by considering twice continuously differentiable desired position trajectories whose 1st and 2nd time-derivative (*i.e.* velocity and acceleration) vectors are uniformly bounded, the computed (special form of the) system dynamics turns out to be bounded. Further, the P and D gains are applied to sigmoidal functions — specifically, the hyperbolic tangent— of the closed loop error variables, giving rise to bounded nonlinear P and D terms. Moreover, an auxiliary (internal) dynamical subsystem is considered for the asymptotic estimation of the system velocity error variables. Consequently, only position measurements are involved in the developed algorithm. In a frictionless setting, such a control scheme was proven to semi-globally stabilize the closed-loop system solutions towards suitable trajectories.

By considering viscous friction in the open-loop dynamics, a globally stabilizing version of the control law in [9] was achieved in [10]. The developed scheme keeps the structure of the controller in [9], but the viscous friction force vector is added to the computed robot dynamics, replacing the current velocity vector by the desired velocity trajectory. Under such considerations, global tracking is achieved for suitable trajectories.

Two alternative dynamical approaches were proposed in [11]. Both consider P and D correction terms where the hyperbolic tangent of the tracking error and filtered tracking error variables, respectively, are involved. The first one relaxes the system parameter dependence by including a bounded adaptive compensation of the robot dynamics, but involves position and velocity measurements. The second one, on the contrary, is free of velocity measurements, keeping a Computed-Torque-like structure, which depends on the exact knowledge of the system parameters. It considers the same form of the gravity, viscous friction, and Coriolis and centrifugal calculated force vectors used in [10], but a special form of inertial (complemented) force vector where the bounded nonlinear P and D terms are included. Semi-global tracking is achieved by both controllers.

More recently, revisited versions of the controller in [10] have been developed in [12] and [13]. In the first of these works, [12], gains scaling the argument of the hyperbolic tangents are incorporated. In the second one, [13], the hyperbolic tangents are replaced by a more general class of saturating functions. In both works, local exponential stability was proved through singular perturbation theory. Contrarily to the previously mentioned works, the developed algorithms were experimentally tested and compared to other bounded and unbounded schemes (being [13] more exhaustive at this point).

Let us note that by the way the bounded nonlinear P and D terms are defined in the previous works, the P and D gains are tied to satisfy a *saturation-avoidance* inequality (since these define the bounds of the P and D terms). Consequently, such control gains cannot take *any* (positive) value, which restricts their *performance-adjustment* natural role. Let us further note that the above-cited works do not completely characterize the class of *desired trajectories* that may be globally tracked through their proposed algorithms.

In this work, a globally stabilizing output feedback controller for the trajectory tracking of robot manipulators with bounded inputs is proposed. It includes saturating-proportional (SP) and saturating-*computed*-derivative (SD_c) correction terms *plus* (+) the continuous calculation of a special form of the robot dynamics (actually the same one used in [10]) —in view of which we refer to the proposed algorithm as the $\mathbf{SP\text{-}SD_c+}$ controller— as well as an auxiliary dynamical subsystem whose output is fed back as an estimation of the system velocity error variables. It may be seen as an improved generalization of the scheme proposed in [10]. As a matter of fact, it is not defined using a specific sigmoidal function, but any one on a set of *saturation* functions. Consequently, the proposed scheme actually constitutes a family of globally stabilizing output feedback bounded controllers. Furthermore, the bound of such saturation functions is explicitly considered in their definition. These are consequently applied to the whole linear P and D expressions, giving the P and D gains the liberty to adopt any positive value. Such a freedom to select any combination of control gains together with the *generalized* saturation function formulation give rise to an infinite variety of possibilities to adjust or improve the closed-loop performance. Further, a class of desired trajectories that may be globally tracked avoiding input saturation and excluding velocity measurements is completely characterized. Global asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory is proved through a strict Lyapunov function. This does not only give an interesting analytical character to the proposed contribution but it also states an important difference with the work in [10] where a stability proof was not developed. The efficiency of the proposed scheme is corroborated through experimental tests on a 2-DOF robot manipulator.

The work is organized as follows. Section II states the general n-DOF serial rigid robot manipulator open-loop dynamics and some of its main properties, as well as considerations and definitions that are involved throughout the study. In Section III, the proposed controller is presented. Section IV states the main result, where the stability analysis is developed and the control objective is proved to be achieved. Experimental results are presented in Section V. Finally, conclusions are given in Section VI.

II. Preliminaries

The following notation is used throughout the paper. $\mathbb{R}_+$ denotes the set of nonnegative real numbers and $\mathbb{R}^n_+$ represents the set of n-dimensional vectors whose elements are nonnegative real numbers. We denote 0_n the origin of $\mathbb{R}^n$, and I_n the $n \times n$ identity matrix. Let $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times m}$. x_i represents the i^{th} element of x. $\|\cdot\|$ stands for the standard Euclidean vector norm and induced matrix norm, *i.e.* $\|x\| = \left[\sum_{i=1}^n x_i^2\right]^{1/2}$ and $\|A\| = \left[\lambda_{\max}(A^TA)\right]^{1/2}$, where $\lambda_{\max}(A^TA)$ represents the maximum eigenvalue of A^TA. Let $\mathcal{A}$ and $\mathcal{E}$ be subsets (with non-empty interior) of some vector spaces $\mathbb{A}$ and $\mathbb{E}$ respectively. We denote $\mathcal{C}^m(\mathcal{A};\mathcal{E})$ the set of m-times continuously differentiable functions from $\mathcal{A}$ to $\mathcal{E}$ (with differentiability at any point on the boundary of $\mathcal{A}$, when included in the set, meant as the limit from the interior of $\mathcal{A}$). Consider a continuous-time function $h \in \mathcal{C}^2(\mathbb{R}_+;\mathcal{E})$. The time-derivative and second-time-derivative of h are respectively represented as $\dot{h}$ and $\ddot{h}$, *i.e.* $\dot{h} : t \mapsto \frac{d}{dt}h$ and $\ddot{h} : t \mapsto \frac{d^2}{dt^2}h$.

Let us consider the general n-DOF serial rigid robot manipulator dynamics with viscous friction [14, §6.2], [15, §2.1]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where $q,\ \dot{q},\ \ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity and acceleration vectors, $D(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with F being a constant, positive definite, diagonal (viscous friction coefficient) matrix, *i.e.* $F = \mathrm{diag}[f_1,\ldots,f_n]$, with $f_i > 0$, $\forall i \in \{1,\ldots,n\}$. The terms of such a dynamical model satisfy some well-known properties (see for instance [1, Chap. 4]). Some of them are recalled here.

Property 1 *The inertia matrix $D(q)$ is a positive definite symmetric matrix satisfying $d_mI_n \le D(q) \le d_MI_n$, $\forall q \in \mathbb{R}^n$, for some positive constants $d_m \le d_M$.*

Property 2 *The Coriolis matrix $C(q,\dot{q})$ satisfies:*

2.1. $x^T\left[\frac{1}{2}\dot{D}(q,\dot{q}) - C(q,\dot{q})\right]x = 0$, $\forall x, q, \dot{q} \in \mathbb{R}^n$;
2.2. $\dot{D}(q,\dot{q}) = C(q,\dot{q}) + C^T(q,\dot{q})$, $\forall q, \dot{q} \in \mathbb{R}^n$;
2.3. $C(w, x+y)z = C(w,x)z + C(w,y)z$, $\forall w, x, y, z \in \mathbb{R}^n$;
2.4. $C(x,y)z = C(x,z)y$, $\forall x,y,z \in \mathbb{R}^n$;
2.5. $\|C(x,y)z\| \le k_c\|y\|\|z\|$, $\forall x,y,z \in \mathbb{R}^n$, *for some constant* $k_c \ge 0$.

Property 3 *The gravity vector satisfies $\|g(q)\| \le \gamma$, $\forall q \in \mathbb{R}^n$, for some positive constant γ, or equivalently, every element of the gravity vector, $g_i(q), i = 1,\ldots,n$, satisfies $|g_i(q)| \le \gamma_i$, $\forall q \in \mathbb{R}^n$, for some positive constants $\gamma_i, i = 1,\ldots,n$.*

Property 4 *The viscous friction coefficient matrix satisfies $f_m\|x\|^2 \le x^TFx \le f_M\|x\|^2$, $\forall x \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \le \max_i\{f_i\} \triangleq f_M$.*

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \leq T_i$, $i = 1, \ldots, n$. In other words, if u_i represents the control signal (controller output) relative to the i^{th} DOF, then

$$\tau_i = T_i \text{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i = 1, \ldots, n$, where $\text{sat}(\cdot)$ is the standard saturation function, *i.e.* $\text{sat}(\varsigma) = \text{sign}(\varsigma)\min\{|\varsigma|, 1\}$.

The control scheme proposed in this work involves a special type of (saturation) functions satisfying the following definition.

Definition 1 *Given a positive constant M, a function $\sigma : \mathbb{R} \to \mathbb{R} : \varsigma \mapsto \sigma(\varsigma)$ is said to be a* ***generalized saturation*** *with bound M, if it is locally Lipschitz, nondecreasing, and satisfies:*

1. $\varsigma\sigma(\varsigma) > 0, \forall \varsigma \neq 0$;
2. $|\sigma(\varsigma)| \leq M, \forall \varsigma \in \mathbb{R}$.

A strictly increasing continuously differentiable function fulfilling Definition 1 has the following properties.

Lemma 1 *Let $\sigma : \mathbb{R} \to \mathbb{R} : \varsigma \mapsto \sigma(\varsigma)$ be a strictly increasing continuously differentiable generalized saturation function with bound M, k and k_0 be positive constants, and $\sigma' : \varsigma \mapsto \frac{d\sigma}{d\varsigma}$. Then*

1. $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) = 0$;
2. $\sigma'(\varsigma)$ *is positive and bounded*, i.e. *there exists a constant* $\sigma'_M \in (0, \infty)$ *such that* $0 < \sigma'(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$;
3. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall \varsigma \in \mathbb{R}$;
4. $\int_0^\varsigma \sigma(kr)dr > 0, \forall \varsigma \neq 0$;
5. $\int_0^\varsigma \sigma(kr)dr \to \infty$ *as* $|\varsigma| \to \infty$;
6. $|\sigma(kx + k_0y) - \sigma(k_0y)| \leq \sigma'_M k|x|, \forall x, y \in \mathbb{R}$.
7. $|\sigma(kx)| \leq \sigma'_M k|x|, \forall x \in \mathbb{R}$.

Proof.

1. Since σ is a continuous function that keeps the sign of its argument (according to item 1 of Definition 1), and is strictly increasing and bounded by M, there exists a positive constant $c \leq M$ such that $\lim_{|\varsigma|\to\infty} |\sigma(\varsigma)| = c$, or equivalently $\lim_{|\varsigma|\to\infty} \sigma(\varsigma) = c \cdot \text{sign}(\varsigma)$. Hence, we have that

$$\begin{aligned}\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) &= \lim_{|\varsigma|\to\infty} \lim_{h\to 0} \frac{\sigma(\varsigma + h) - \sigma(\varsigma)}{h} \\ &= \lim_{h\to 0} \lim_{|\varsigma|\to\infty} \frac{\sigma(\varsigma + h) - \sigma(\varsigma)}{h} \\ &= \lim_{h\to 0} \frac{c \cdot \text{sign}(\varsigma) - c \cdot \text{sign}(\varsigma)}{h} = 0\end{aligned}$$

2. Since σ is a continuously differentiable and strictly increasing function, we have that $\sigma'(\varsigma)$ exists and is continuous on $\mathbb{R}$, and $\sigma'(\varsigma) > 0$, $\forall \varsigma \in \mathbb{R}$. Furthermore, in view of its continuity, $\sigma'(\varsigma)$ is bounded on any compact subset of $\mathbb{R}$. Thus, its boundedness will be uniform if $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) < \infty$. Since $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) = 0$, according to item 1 of the Lemma, we conclude that $\sigma'(\varsigma)$ is uniformly bounded, *i.e.* $\exists \sigma'_M > 0$ such that $\sigma'(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$.
3. From continuous differentiability —implying Lipschitz-continuity— of σ and item 2 of the Lemma, it follows that

$$\frac{\sigma'(k\varsigma)}{\sigma'_M} |\sigma(k\varsigma)| \leq |\sigma(k\varsigma)| \leq \sigma'_M |k\varsigma|$$

$\forall \varsigma \in \mathbb{R}$, wherefrom, considering that σ has the sign of its argument (according to item 1 of Definition 1), we have that

$$\int_0^\varsigma \frac{\sigma(kr)}{\sigma'_M}\sigma'(kr)dr \leq \int_0^\varsigma \sigma(kr)dr \leq \int_0^\varsigma \sigma'_M krdr$$

wherefrom we get

$$\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$$

$\forall \varsigma \in \mathbb{R}$.
4. Strict positivity of $\int_0^\varsigma \sigma(kr)dr$ on $\mathbb{R} \setminus \{0\}$ follows from item 3 of the Lemma, by noting (from item 1 of Definition 1) that $\sigma^2(k\varsigma) > 0$, $\forall \varsigma \neq 0$.
5. From the continuous differentiability and strictly increasing characters of σ, and its satisfaction of item 2 of the Lemma, we have that $\sigma'(k\varsigma)$ is continuous, positive, and bounded on $\mathcal{I}_a \triangleq [-a, a]$, for any $a > 0$, in such a way that

$$0 < \inf_{r\in\mathcal{I}_a} \sigma'(kr) \leq \sigma'(k\varsigma) \leq \sup_{r\in\mathcal{I}_a} \sigma'(kr) \leq \sigma'_M \tag{3}$$

$\forall \varsigma \in \mathcal{I}_a$. Let us consider a positive constant $k_a \leq \inf_{r\in\mathcal{I}_a} \sigma'(kr)$. Then, from (3), we have that

$$\left|k_a a \,\text{sat}\left(\frac{\varsigma}{a}\right)\right| \leq |\sigma(k\varsigma)|$$

$\forall \varsigma \in \mathbb{R}$, wherefrom we get

$$S_a(\varsigma) = \int_0^\varsigma k_a a \,\text{sat}\left(\frac{r}{a}\right) dr \leq \int_0^\varsigma \sigma(k\varsigma)dr$$

$\forall \varsigma \in \mathbb{R}$, with

$$S_a(\varsigma) \triangleq \begin{cases} \frac{k_a}{2}\varsigma^2 & \forall |\varsigma| \leq a \\ k_a a\left(|\varsigma| - \frac{a}{2}\right) & \forall |\varsigma| > a \end{cases}$$

Thus, from these expressions we observe, on the one hand, that $\lim_{|\varsigma|\to\infty} S_a(\varsigma) \le \lim_{|\varsigma|\to\infty} \int_0^\varsigma \sigma(kr)dr$, and, on the other, that $S_a(\varsigma) \to \infty$ as $|\varsigma| \to \infty$, wherefrom we conclude that $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$.

6. Let $w, x, y, z \in \mathbb{R}$. From continuous differentiability of σ and item 2 of the Lemma, we have that σ satisfies the Lipschitz condition globally on $\mathbb{R}$ with σ'_M as Lipschitz constant (see for instance [16, Lemma 3.3]), *i.e.* $|\sigma(w) - \sigma(z)| \le \sigma'_M |w - z|$, $\forall w, z \in \mathbb{R}$. By taking $w = kx + k_0 y$ and $z = k_0 y$, we get $|\sigma(kx + k_0 y) - \sigma(k_0 y)| \le \sigma'_M k|x|$, $\forall x, y \in \mathbb{R}$.
7. From item 6 of the Lemma with $y = 0$, we have that $|\sigma(kx)| \le \sigma'_M k|x|$, $\forall x \in \mathbb{R}$.

$\square$

We state the **control objective** as the global stabilization of the robot configuration vector variable, q, towards a (suitable) desired trajectory vector, $q_d(t)$, through a bounded control scheme that only feeds back configuration variables from the robot —consequently disregarding time-derivatives of q of any order— and avoids input saturations *i.e.* such that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \dots, n$, $\forall t \ge 0$ (see (2)).

III. Proposed Controller

The following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1 $T_i > \gamma_i$, $\forall i \in \{1, \dots, n\}$.

Further, in order to guarantee the achievement of the stated control objective, the proposed scheme is restricted to desired trajectory vectors meeting the following:

Assumption 2 *The desired trajectory vector $q_d(t)$ is a twice continuously differentiable function —i.e. $q_d \in \mathcal{C}^2(\mathbb{R}_+; \mathbb{R}^n)$— satisfying*

$$\sup_{t\ge 0} \|\dot{q}_d(t)\| \le B_{dv} \tag{4a}$$

and

$$\sup_{t\ge 0} \|\ddot{q}_d(t)\| \le B_{da} \tag{4b}$$

for some (desired velocity and acceleration vector) bounds such that

$$(B_{dv}, B_{da}) \in \mathcal{B}_1 \cup \mathcal{B}_2$$

where

$$\mathcal{B}_i \triangleq \{(\xi, \zeta) \in \mathbb{R}_+^2 \mid \xi < B_{vi}\,,\ \zeta < B_{ai}\}$$

$i = 1, 2,$

$$B_{v1} \triangleq \begin{cases} \min\left\{ \frac{f_m}{k_c}\,,\ B_{11} \right\} & \text{if } k_c > 0 \\ B_{10} & \text{if } k_c = 0 \end{cases} \tag{5a}$$

$$B_{a1} \triangleq \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} \tag{5b}$$

$$B_{11} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c}} \tag{5c}$$

$$B_{10} \triangleq \frac{\Delta_m}{f_M} \tag{5d}$$

and

$$B_{a2} \triangleq \frac{\Delta_m}{d_M} \tag{6a}$$

$$B_{v2} \triangleq \begin{cases} \min\left\{ \frac{f_m}{k_c}\,,\ B_{21} \right\} & \text{if } k_c > 0 \\ B_{20} & \text{if } k_c = 0 \end{cases} \tag{6b}$$

$$B_{21} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c}} \tag{6c}$$

$$B_{20} \triangleq \frac{\Delta_m - d_M B_{da}}{f_M} \tag{6d}$$

with

$$\Delta_m \triangleq \min_i \{T_i - \gamma_i\} \tag{7}$$

Under Assumptions 1 and 2, we propose a control scheme —which we refer to as SP-SD$_c$+ controller— of the form

$$u = -s_2(K_2\bar{q}) - s_1(K_1\vartheta) + \tau_c(q, \dot{q}_d, \ddot{q}_d) \tag{8a}$$

where

$$\tau_c(q, \dot{q}_d, \ddot{q}_d) = D(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + F\dot{q}_d + g(q)$$

$\bar{q} \triangleq q - q_d(t)$; K_1 and K_2 are positive definite diagonal matrices, *i.e.* $K_1 = \text{diag}\,[k_{11}, \dots, k_{1n}]$ and $K_2 = \text{diag}\,[k_{21}, \dots, k_{2n}]$ with $k_{1i} > 0$ and $k_{2i} > 0$ for all $i = 1, \dots, n$; ϑ is the output of an auxiliary (internal) dynamical subsystem defined as follows

$$\dot{q}_c = -AK_1^{-1} s_1\big(K_1(q_c + B\bar{q})\big) \tag{8b}$$

$$\vartheta = q_c + B\bar{q} \tag{8c}$$

with A and B being positive definite diagonal matrices, *i.e.* $A = \text{diag}\,[a_1, \dots, a_n]$ and $B = \text{diag}\,[b_1, \dots, b_n]$

with $a_i > 0$ and $b_i > 0$ for all $i = 1, \ldots, n$; and $s_j : \mathbb{R}^n \to \mathbb{R}^n : x \mapsto s_j(x) = \big(\sigma_{j1}(x_1), \ldots, \sigma_{jn}(x_n)\big)^T$, $j = 1, 2$, with $\sigma_{ji}(\cdot)$, $i = 1, \ldots, n$, being **strictly increasing continuously differentiable generalized saturation functions** with bounds M_{ji} satisfying

$$M_{1i} + M_{2i} \leq T_i - d_M B_{da} - k_c B_{dv}^2 - f_i B_{dv} - \gamma_i \tag{9}$$

(see Properties 1, 2.5, 3, and 4), $\forall i = 1, \ldots, n$. Note that $\dot{q}$ is not involved in any of the expressions in Eqs. (8).

Remark 1 *Under inequalities (9), input saturation is avoided globally in time, as will be shown in Section IV below. In this direction, let us note that the satisfaction of Assumption 2 guarantees the existence of positive values M_{1i} and M_{2i} fulfilling (9). In turn, Assumption 1 renders possible the* tractable *desired trajectory characterization stated by Assumption 2. Indeed, observe on the one hand that, under the satisfaction of Assumption 1, we have $\Delta_m > 0$ (see (7)), which implies $B_{11} > 0$ (see (5c)) and $B_{10} > 0$ (see (5d)), which in turn entail $B_{v1} > 0$ (see (5a)), which renders possible to state some positive value $B_{dv} < B_{v1}$. With such a value of B_{dv}, we have, if $k_c > 0$: $B_{dv} < -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c}} \implies \left(B_{dv} + \frac{f_M}{2k_c}\right)^2 < \left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c} \implies B_{dv}^2 + \frac{f_M B_{dv}}{k_c} < \frac{\Delta_m}{k_c} \implies \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} > 0$, i.e. $B_{a1} > 0$ (see (5b)), or similarly, if $k_c = 0$: $B_{dv} < \frac{\Delta_m}{f_M} \implies \frac{\Delta_m - f_M B_{dv}}{d_M} > 0$, or equivalently, for any $k_c \geq 0$: $\frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} > 0$, i.e. $B_{a1} > 0$, which makes possible to state some positive value $B_{da} < B_{a1}$, by virtue of which $\mathcal{B}_1$ is non-empty. On the other hand, observe that, under the satisfaction of Assumption 1, we have $\Delta_m > 0$ (see (7)), which implies $B_{a2} > 0$ (see (6a)), which renders possible to state some positive value $B_{da} < B_{a2}$. With such a value of B_{da}, we have $B_{da} < \frac{\Delta_m}{d_M} \implies \Delta_m - d_M B_{da} > 0$, which implies $B_{20} > 0$ (see (6d)) and $B_{21} > 0$ (see (6c)), which in turn entail $B_{v2} > 0$ (see (6b)), which makes possible to state some positive value $B_{dv} < B_{v2}$, by virtue of which $\mathcal{B}_2$ is non-empty. Thus, the satisfaction of Assumption 1 renders possible to choose a desired trajectory q_d fulfilling Assumption 2. Further, observe that with a value of $B_{da} < B_{a1}$, we have $B_{da} < \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} \implies \min_i\{T_i - \gamma_i\} - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - k_c B_{dv}^2 - f_i B_{dv} - \gamma_i > 0, \forall i = 1, \ldots, n$, ensuring positivity of the right-hand-side expression of inequalities (9), while with a value of $B_{dv} < B_{v2}$ we have, if $k_c > 0$: $B_{dv} < -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c}} \implies \left(B_{dv} + \frac{f_M}{2k_c}\right)^2 < \left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c} \implies B_{dv}^2 + \frac{f_M B_{dv}}{k_c} < \frac{\Delta_m - d_M B_{da}}{k_c} \implies \min_i\{T_i - \gamma_i\} - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - k_c B_{dv}^2 - f_i B_{dv} - \gamma_i > 0$, $\forall i = 1, \ldots, n$, or similarly if $k_c = 0$: $B_{dv} < \frac{\Delta_m - d_M B_{da}}{f_M} \implies \min_i\{T_i - \gamma_i\} - d_M B_{da} - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - f_i B_{dv} - \gamma_i > 0$, $\forall i = 1, \ldots, n$, or equivalently, for any $k_c \geq 0$: $T_i - d_M B_{da} - k_c B_{dv}^2 - f_i B_{dv} - \gamma_i > 0$, $\forall i = 1, \ldots, n$, ensuring positivity of the right-hand-side expression of inequalities (9) in this case too. Thus, the satisfaction of Assumption 2 indeed guarantees the existence of positive values M_{1i} and M_{2i} fulfilling (9).*

IV. Main Result

Proposition 1 *Consider the system (1)–(2) with the control law in Eqs. (8) under Assumptions 1 and 2 and the satisfaction of inequalities (9). For any positive definite diagonal matrices K_1, K_2, A, and B, global uniform asymptotic stabilization of the closed-loop system solutions $q(t)$ towards the desired trajectory vector $q_d(t)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.*

Proof. From (8a), (9), Properties 1, 2.5, 3, and 4, and the strict increasing character of the involved generalized saturation functions, one sees that $|u_i(t)| < M_{1i} + M_{2i} + d_M B_{da} + k_c B_{dv}^2 + f_i B_{dv} + \gamma_i \leq T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. From this and (2) it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. We now focus on the stability analysis. The closed-loop dynamics takes the form

$$\begin{aligned} D(q)\ddot{\bar{q}} + \left[C(q, \dot{q}) + C\big(q, \dot{q}_d(t)\big)\right]\dot{\bar{q}} + F\dot{\bar{q}} \\ + s_1(K_1\vartheta) + s_2(K_2\bar{q}) = 0_n \end{aligned} \tag{10a}$$

$$\dot{\vartheta} = -AK_1^{-1}s_1(K_1\vartheta) + B\dot{\bar{q}} \tag{10b}$$

where Property 2.4 has been used (observe from the definition of $\bar{q}$, stated in Section III, that $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$).* Let us define the scalar function

$$\begin{aligned} V(t, \bar{q}, \dot{\bar{q}}, \vartheta) = \frac{1}{2}\dot{\bar{q}}^T D(q)\dot{\bar{q}} + \int_{0_n}^{\bar{q}} s_2^T(K_2 r)dr \\ + \int_{0_n}^{\vartheta} s_1^T(K_1 r)B^{-1}dr \\ + \varepsilon\dot{\bar{q}}^T D(q)\left[s_2(K_2\bar{q}) - s_1(K_1\vartheta)\right] \end{aligned} \tag{11}$$

*For the sake of simplicity, throughout the proof, $D(q)$ and $C(q, \cdot)$ will be used instead of $D\big(\bar{q} + q_d(t)\big)$ and $C\big(\bar{q} + q_d(t), \cdot\big)$.

where $\int_{0_n}^{\bar{q}} s_2^T(K_2r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{2i}(k_{2i}r_i)dr_i$, $\int_{0_n}^{\vartheta} s_1^T(K_1r)dr = \sum_{i=1}^n \int_0^{\vartheta_i} \sigma_{1i}(k_{1i}r_i)dr_i$, and ε is a positive constant satisfying[†]

$$\varepsilon < \min\left\{ \sqrt{\frac{\alpha}{k_{2M}\sigma'_{2M}}}\ , \sqrt{\frac{\alpha}{b_M k_{1M}\sigma'_{1M}}}\ , \frac{a_m}{2b_M k_{1M}}\ , \frac{f_m - k_c B_{dv}}{2\left(k_c B_M + d_M k_{2M}\sigma'_{2M}\right) + \left(2k_c B_{dv} + f_M\right)^2}\ , \sqrt{\frac{a_m(f_m - k_c B_{dv})}{b_M k_{1M}\left(2k_c B_d + f_M + a_M d_M \sigma'_{1M}\right)^2}} \right\} \tag{12}$$

with $\alpha \triangleq \frac{d_m}{4d_M^2}$, $\sigma'_{jM} \triangleq \max_i\{\sigma'_{jiM}\}$ (see item 2 of Lemma 1) and $k_{jM} \triangleq \max_i\{k_{ji}\}$, $j = 1, 2$, and $B_M \triangleq \sum_{j=1}^2 \sqrt{\sum_{i=1}^n M_{ji}^2}$. Let us note, from Property 1 and items 3 and 7 of Lemma 1, that

$$W_{11}(\bar{q},\dot{\bar{q}}) + W_{12}(\dot{\bar{q}},\vartheta) \le V(t,\bar{q},\dot{\bar{q}},\vartheta) \le W_{21}(\bar{q},\dot{\bar{q}}) + W_{22}(\dot{\bar{q}},\vartheta)$$

with

$$W_{11}(\bar{q},\dot{\bar{q}}) \triangleq \frac{d_m}{4}\|\dot{\bar{q}}\|^2 + \frac{1}{2}\int_{0_n}^{\bar{q}} s_2^T(K_2r)dr + \frac{\|s_2(K_2\bar{q})\|^2}{4k_{2M}\sigma'_{2M}} - \varepsilon d_M\|\dot{\bar{q}}\|\|s_2(K_2\bar{q})\|$$

$$W_{12}(\dot{\bar{q}},\vartheta) \triangleq \frac{d_m}{4}\|\dot{\bar{q}}\|^2 + \frac{1}{2}\int_{0_n}^{\vartheta} s_1^T(K_1r)B^{-1}dr + \frac{\|s_1(K_1\vartheta)\|^2}{4b_M k_{1M}\sigma'_{1M}} - \varepsilon d_M\|\dot{\bar{q}}\|\|s_1(K_1\vartheta)\|$$

$$W_{21}(\bar{q},\dot{\bar{q}}) \triangleq \frac{d_M}{4}\|\dot{\bar{q}}\|^2 + \frac{k_{2M}\sigma'_{2M}}{2}\|\bar{q}\|^2 + \varepsilon d_M k_{2M}\sigma'_{2M}\|\bar{q}\|\|\dot{\bar{q}}\|$$

$$W_{22}(\dot{\bar{q}},\vartheta) \triangleq \frac{d_M}{4}\|\dot{\bar{q}}\|^2 + \frac{k_{1M}\sigma'_{1M}}{2b_m}\|\vartheta\|^2 + \varepsilon d_M k_{1M}\sigma'_{1M}\|\dot{\bar{q}}\|\|\vartheta\|$$

[†]Observe that the satisfaction of Assumption 2 guarantees positivity of the fourth term within the braces in (12) and of the expression within the square root of the fifth term, consequently ensuring the existence of a positive ε fulfilling inequality (12). Indeed, note that for a desired trajectory with velocity vector bound such that $B_{dv} < B_{v1}$ or $B_{dv} < B_{v2}$, we have (see (5a) and (6b)), if $k_c > 0$: $B_{dv} < \frac{f_m}{k_c} \implies f_m - k_c B_{dv} > 0$. From this and the consideration of Property 4, we have, for any $k_c \ge 0$: $f_m - k_c B_{dv} > 0$, wherefrom positivity of the fourth term within the braces in (12), and of the expression within the square root of the fifth term, is guaranteed.

Moreover, note that these functions may be rewritten as

$$W_{11}(\bar{q},\dot{\bar{q}}) = \frac{1}{2}\int_{0_n}^{\bar{q}} s_2^T(K_2r)dr + \frac{1}{4}\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{11}\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$W_{12}(\dot{\bar{q}},\vartheta) = \frac{1}{2}\int_{0_n}^{\vartheta} s_1^T(K_1r)B^{-1}dr + \frac{1}{4}\begin{pmatrix}\|\dot{\bar{q}}\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}^T P_{12}\begin{pmatrix}\|\dot{\bar{q}}\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}$$

$$W_{21}(\bar{q},\dot{\bar{q}}) = \frac{1}{2}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{21}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$W_{22}(\dot{\bar{q}},\vartheta) = \frac{1}{2}\begin{pmatrix}\|\dot{\bar{q}}\| \\ \|\vartheta\|\end{pmatrix}^T P_{22}\begin{pmatrix}\|\dot{\bar{q}}\| \\ \|\vartheta\|\end{pmatrix}$$

where

$$P_{11} = \begin{pmatrix}\frac{1}{k_{2M}\sigma'_{2M}} & -2\varepsilon d_M \\ -2\varepsilon d_M & d_m\end{pmatrix}$$

$$P_{12} = \begin{pmatrix}d_m & -2\varepsilon d_M \\ -2\varepsilon d_M & \frac{1}{b_M k_{1M}\sigma'_{1M}}\end{pmatrix}$$

$$P_{21} = \begin{pmatrix}k_{2M}\sigma'_{2M} & \varepsilon d_M k_{2M}\sigma'_{2M} \\ \varepsilon d_M k_{2M}\sigma'_{2M} & \frac{d_M}{2}\end{pmatrix}$$

$$P_{22} = \begin{pmatrix}\frac{d_M}{2} & \varepsilon d_M k_{1M}\sigma'_{1M} \\ \varepsilon d_M k_{1M}\sigma'_{1M} & \frac{k_{1M}\sigma'_{1M}}{b_m}\end{pmatrix}$$

Further, since $\varepsilon < \min\left\{\sqrt{\frac{\alpha}{k_{2M}\sigma'_{2M}}}\ , \sqrt{\frac{\alpha}{b_M k_{1M}\sigma'_{1M}}}\right\}$ (see (12)), one can verify (after several basic developments) that P_{11}, P_{12}, P_{21}, and P_{22} are positive definite symmetric matrices. From this and items 4 and 5 of Lemma 1, one sees that $V(t,\bar{q},\dot{\bar{q}},\vartheta)$ is positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned}\dot{V}(t,\bar{q},\dot{\bar{q}},\vartheta) = &-\dot{\bar{q}}^T C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} \\ &- s_1^T(K_1\vartheta)B^{-1}AK_1^{-1}s_1(K_1\vartheta) \\ &- \varepsilon s_2^T(K_2\bar{q})C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - \varepsilon s_2^T(K_2\bar{q})F\dot{\bar{q}} \\ &- \varepsilon s_2^T(K_2\bar{q})s_2(K_2\bar{q}) + \varepsilon s_1^T(K_1\vartheta)C(q,\dot{\bar{q}})\dot{\bar{q}} \\ &+ \varepsilon s_1^T(K_1\vartheta)F\dot{\bar{q}} + \varepsilon s_1^T(K_1\vartheta)s_1(K_1\vartheta) \\ &+ \varepsilon\dot{\bar{q}}^T C(q,\dot{\bar{q}})\big[s_2(K_2\bar{q}) - s_1(K_1\vartheta)\big] \\ &+ \varepsilon\dot{\bar{q}}^T C\big(q,\dot{q}_d(t)\big)\big[s_2(K_2\bar{q}) - s_1(K_1\vartheta)\big] \\ &+ \varepsilon\dot{\bar{q}}^T D(q)s_2'(K_2\bar{q})K_2\dot{\bar{q}} \\ &+ \varepsilon\dot{\bar{q}}^T D(q)s_1'(K_1\vartheta)\big[As_1(K_1\vartheta) - K_1B\dot{\bar{q}}\big]\end{aligned}$$

with $s_2'(K_2\bar{q}) = \text{diag}\left[\sigma'_{21}(k_{21}\bar{q}_1), \ldots, \sigma'_{2n}(k_{2n}\bar{q}_n)\right]$ and $s_1'(K_1\vartheta) = \text{diag}\left[\sigma'_{11}(k_{11}\vartheta_1), \ldots, \sigma'_{1n}(k_{1n}\vartheta_n)\right]$, where

$D(q)\ddot{q}$ has been replaced by its equivalent expression from the closed-loop dynamics (10a), and Properties 2.1–2.3 have been used. From Properties 1, 2.5 and 4, and the satisfaction of inequality (4a), we have that

$$\dot{V}(t,\bar{q},\dot{\bar{q}},\vartheta) \leq W_{31}(\bar{q},\dot{\bar{q}}) + W_{32}(\bar{q},\vartheta) + W_{33}(\dot{\bar{q}},\vartheta)$$

with

$$\begin{aligned} W_{31}(\bar{q},\dot{\bar{q}}) \triangleq\ & \frac{k_c B_{dv}}{2}\|\dot{\bar{q}}\|^2 - \frac{f_m}{2}\|\dot{\bar{q}}\|^2 \\ & + \varepsilon k_c B_{dv}\|\dot{\bar{q}}\|\|s_2(K_2\bar{q})\| \\ & + \varepsilon f_M\|\dot{\bar{q}}\|\|s_2(K_2\bar{q})\| - \frac{\varepsilon}{2}\|s_2(K_2\bar{q})\|^2 \\ & + \varepsilon B_{2M}k_c\|\dot{\bar{q}}\|^2 + \varepsilon k_c B_{dv}\|\dot{\bar{q}}\|\|s_2(K_2\bar{q})\| \\ & + \varepsilon B_{1M}k_c\|\dot{\bar{q}}\|^2 + \varepsilon d_M\sigma'_{2M}k_{2M}\|\dot{\bar{q}}\|^2 \\ W_{32}(\bar{q},\vartheta) \triangleq\ & -\frac{a_m}{2b_M k_{1M}}\|s_1(K_1\vartheta)\|^2 - \frac{\varepsilon}{2}\|s_2(K_2\bar{q})\|^2 \\ & + \varepsilon\|s_1(K_1\vartheta)\|^2 \\ W_{33}(\dot{\bar{q}},\vartheta) \triangleq\ & \frac{k_c B_{dv}}{2}\|\dot{\bar{q}}\|^2 - \frac{f_m}{2}\|\dot{\bar{q}}\|^2 \\ & - \frac{a_m}{2b_M k_{1M}}\|s_1(K_1\vartheta)\|^2 \\ & + \varepsilon k_c B_{dv}\|\dot{\bar{q}}\|\|s_1(K_1\vartheta)\| \\ & + \varepsilon f_M\|\dot{\bar{q}}\|\|s_1(K_1\vartheta)\| \\ & + \varepsilon k_c B_{dv}\|\dot{\bar{q}}\|\|s_1(K_1\vartheta)\| \\ & + \varepsilon d_M\sigma'_{1M}a_M\|\dot{\bar{q}}\|\|s_1(K_1\vartheta)\| \end{aligned}$$

where $B_{jM} \triangleq \sqrt{\sum_{i=1}^n M_{ji}^2}$, $j=1,2$, and the facts that $\|s_2(K_2\bar{q})\| \leq B_{2M}$ and $\|s_2'(K_2\bar{q})\| \leq \sigma'_{2M}$, $\forall\bar{q}\in\mathbb{R}^n$, as well as $\|s_1(K_1\vartheta)\| \leq B_{1M}$ and $\|s_1'(K_1\vartheta)\| \leq \sigma'_{1M}$, $\forall\vartheta\in\mathbb{R}^n$, and item 2 of Lemma 1, were considered. Notice that these functions may be rewritten as

$$W_{31}(\bar{q},\dot{\bar{q}}) = -\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_1 \begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$W_{32}(\bar{q},\vartheta) = -\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}^T Q_2 \begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}$$

$$W_{33}(\dot{\bar{q}},\vartheta) = -\begin{pmatrix}\|\dot{\bar{q}}\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}^T Q_3 \begin{pmatrix}\|\dot{\bar{q}}\| \\ \|s_1(K_1\vartheta)\|\end{pmatrix}$$

with

$$Q_1 = \begin{pmatrix} \frac{\varepsilon}{2} & -\varepsilon\left(k_c B_{dv} + \frac{f_M}{2}\right) \\ -\varepsilon\left(k_c B_{dv} + \frac{f_M}{2}\right) & Q_{122}\end{pmatrix}$$

$$Q_2 = \begin{pmatrix}\frac{\varepsilon}{2} & 0 \\ 0 & \frac{a_m}{2b_M k_{1M}} - \varepsilon\end{pmatrix}$$

Fig. 1. Experimental robot arm

$$Q_3 = \begin{pmatrix}\frac{f_m - k_c B_{dv}}{2} & Q_{312} \\ Q_{312} & \frac{a_m}{2b_M k_{1M}}\end{pmatrix}$$

where

$$Q_{122} = \frac{f_m - k_c B_{dv}}{2} - \varepsilon(k_c B_M + d_M k_{2M}\sigma'_{2M})$$

and

$$Q_{312} = -\varepsilon\left(k_c B_{dv} + \frac{f_M + a_M d_M\sigma'_{1M}}{2}\right)$$

Further, since

$$\varepsilon < \min\left\{\frac{a_m}{2b_M k_{1M}}, \frac{f_m - k_c B_{dv}}{2\left(k_c B_M + d_M k_{2M}\sigma'_{2M}\right) + (2k_c B_{dv} + f_M)^2}, \sqrt{\frac{a_m(f_m - k_c B_{dv})}{b_M k_{1M}\left(2k_c B_{dv} + f_M + a_M d_M\sigma'_{1M}\right)^2}}\right\}$$

(see (12)), one can verify (after several basic developments) that Q_1, Q_2, and Q_3 are positive definite symmetric matrices. Hence, $\dot{V}(t,\bar{q},\dot{\bar{q}},\vartheta)$ is negative definite. Thus, from Lyapunov's stability theory (applied to non-autonomous systems; see for instance [16, Theo. 4.9]), the proposition follows. □

V. Experimental results

Aiming at verifying the effectiveness of the proposed controller, real-time experiments were carried out on a well-identified two-axis planar robot arm. This manipulator was built keeping the same mechanical structure of the one described and used in [17]; see Fig. 1. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque sources and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a 2.5 msec sampling period in a control board (based on a DSP 32-bit floating point microprocessor) mounted on a PC host computer.

For the considered experimental manipulator, the various terms characterizing the system dynamics in (1) are given by

$$D(q) = \begin{bmatrix} 2.351 + 0.168\cos q_2 & 0.102 + 0.084\cos q_2 \\ 0.102 + 0.084\cos q_2 & 0.102 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -0.084\dot{q}_2 \sin q_2 & -0.084(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ 0.084\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} 38.465\sin q_1 + 1.825\sin(q_1 + q_2) \\ 1.825\sin(q_1 + q_2) \end{bmatrix}$$

and $F = \mathrm{diag}[2.288, 0.175]$. Thus, Properties 1–4 are satisfied with $d_m = 0.07$ kg m^2, $d_M = 2.5$ kg m^2, $k_c = 0.1422$ kg m^2, $\gamma_1 = 40.29$ Nm, $\gamma_2 = 1.83$ Nm, $f_m = 0.175$ kg m^2/sec, and $f_M = 2.288$ kg m^2/sec. The maximum torques allowed are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. Observe that Assumption 1 is fulfilled.

For comparison purposes, additional experiments were run considering the controller in [10] —referred to as **S-K'01**— which may be seen as a special case of the proposed SP-SD$_c$+ scheme in Eqs. (8) taking $\sigma_{ji}(\varsigma) = k_{ji}\tanh(\lambda\varsigma/k_{ji})$, $\forall (i,j) \in \{1,\dots,n\} \times \{1,2\}$, with $\lambda = 1$ sec, and k_{ji} such that $k_{1i} + k_{2i} \leq T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i, \forall i = 1,\dots,n$ (the *saturation-avoidance* inequalities). The desired trajectory vector, for both controllers, was defined as

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} \triangleq \begin{pmatrix} \frac{\pi}{3} + \sin t \\ \cos t \end{pmatrix} \tag{13}$$

For such a desired trajectory, inequalities (4) are satisfied with $B_{dv} = 1$ rad/sec and $B_{da} = 1$ rad/sec^2. The initial conditions at every test were $q_i(0) = \dot{q}_i(0) = q_{ci}(0) = 0$, $i = 1, 2$. The generalized saturation functions were defined as

$$\sigma_{ji}(\varsigma) = \begin{cases} \rho_{ji}^-(\varsigma) & \text{if } \varsigma < -L_{ji} \\ \varsigma & \text{if } |\varsigma| \leq L_{ji} \\ \rho_{ji}^+(\varsigma) & \text{if } \varsigma > L_{ji} \end{cases}$$

where $\rho_{ji}^{\pm}(\varsigma) = \pm L_{ji} + (M_{ji} - L_{ji})\tanh\left(\frac{\varsigma \mp L_{ji}}{M_{ji} - L_{ji}}\right)$, with $L_{ji} < M_{ji}$, $\forall (i,j) \in \{1,2\} \times \{1,2\}$. The control gains were adjusted in the SP-SD$_c$+ case as: $k_{11} = 150$, $k_{12} = 10$, $k_{21} = 500$, and $k_{22} = 150$ (with k_{1i} in Nm sec and k_{2i} in Nm, $i = 1, 2$); and in the S-K'01 case as: $k_{11} = 14$, $k_{12} = 0.5$, $k_{21} = 90$, and $k_{22} = 7.5$

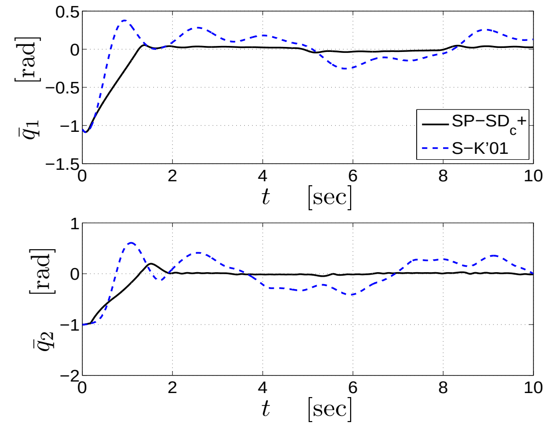

Fig. 2. Position errors

(all of them in Nm);[‡] while, for both controllers, the auxiliary dynamic parameters were fixed as: $a_1 = 150$, $a_2 = 150$, $b_1 = 10$, and $b_2 = 15$ (all of them in sec^{-1}). The saturation function parameters (in the SP-SD$_c$+ case) were defined as: $M_{11} = 65$, $M_{21} = 25$, $M_{12} = 5$, $M_{22} = 2.5$, and $L_{ji} = 0.9M_{ji}$, $\forall (i,j) \in \{1,2\} \times \{1,2\}$ (all of the in Nm). One can easily verify that Assumption 2 as well as inequalities (9) are satisfied.

Fig. 2 shows the evolution of the shoulder and elbow joint position errors, *i.e.* $\bar{q}_1(t)$ and $\bar{q}_2(t)$, and Fig. 3 shows the applied inputs, τ_1 and τ_2, for both tested schemes. Observe that both algorithms avoid input saturation and that the control objective is achieved through the SP-SD$_c$+ controller while disappointing closed loop responses are obtained through the S-K'01 scheme. This highlights the disadvantage of a controller that constrains the control gains to satisfy saturation-avoidance inequalities with respect to one that gives them the liberty to adopt any (positive) value.

VI. Conclusions

In this work, a globally stabilizing output feedback scheme for the trajectory tracking of robot manipulators with bounded inputs was proposed. It achieves the motion control objective avoiding input saturation and excluding velocity measurements. Moreover, it was not defined using a specific sigmoidal function, but any

[‡]In every case, the selected control gain combination was the one giving rise to the best closed loop response obtained after numerous trial-and-error tests. In the S-K'01 case, this tuning procedure was performed by additionally considering the satisfaction of the saturation-avoidance inequalities.

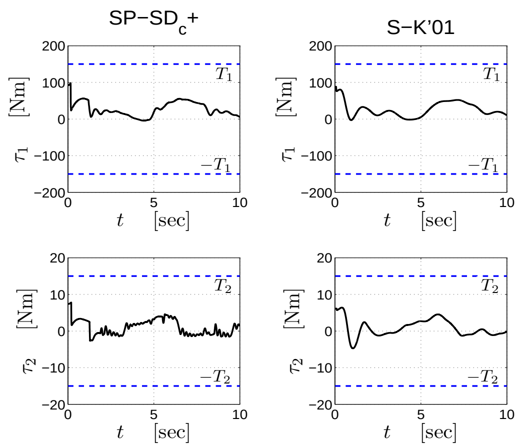

Fig. 3. Control torques

one on a set of *saturation* functions. Consequently, it actually constitutes a family of globally stabilizing output feedback bounded controllers. Such a *generalized* formulation permitted the developed algorithm to adopt a suitable structure where the control gains were able to take any positive value, which may be considered beneficial for performance adjustment/improvement purposes. Furthermore, a class of *desired trajectories* that may be globally tracked avoiding input saturation and excluding velocity measurements was completely characterized. Global asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory was proved through a strict Lyapunov function. The efficiency of the proposed scheme was corroborated through experimental tests carried out on a 2-DOF robot manipulator. The proposed algorithm satisfactorily proved its ability to achieve the tracking control objective avoiding input saturation and without the need of velocity measurements.

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