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Activation of Neuronal Ensembles Via Controlled Synchronization

Gualberto Solís-Perales, and Juan Gonzalo Barajas-Ramírez

Abstract: In this contribution we present the activation of neuronal ensembles of Hindmarsh-Rose neurons by controlled synchronization. The main problem consists in to impose a particular spiking-bursting behavior in all the neurons of the network. We consider a network where the neurons are in its resting state, it is desired that the neurons change their resting state to a particular behavior of activation, dictated by a neuron called the reference neuron. The goal is reached by controlling some neurons in the network controlling only the membrane potential (electrical synapse). The key feature of the present contribution is that by controlling a small number of neurons in the network a desired behavior is induced in all the neurons in the network despite its network topology. The important parameters are the control gain and the coupling strength, thus the activation of the network lays down on a compromise between the control gain and the coupling strength.

Keywords: Neuron Activation, Synchronization, Complex Networks, Control of Complex Networks.

1. INTRODUCTION

In recent years, the study of dynamical networks has attracted a lot of attention, due in part to the many interesting problems and applications in electronics, neural systems, social communities, diseases spreading, and biological systems to mention just a few. Dynamical networks pose many challenges extending from the interplay among their dynamical and structural components until problems on establishing models that capture their key topological features, to determine the stability of their collective behavior [1, 2]. The reader is directed to [3–5] and references therein, for a detailed review of applications and important characteristics and classifications of complex networks problems. For instance, a problem in this context is the synchronization of networks with strictly different nodes, it was reported that through proper coupling modifications a network with nonidentical nodes can achieve synchronous behavior on a common chaotic attractor [6]. An interesting point is that the resulting synchronized collective behavior is determined by the dynamical characteristics of each node, the structure of their interconnections and their coupling strength. These observations indicate that a desired behavior can be induced into a synchronized dynamical network by properly choosing these network features. Then, a controller can be designed to force the entire network to track an arbitrarily chosen reference in a synchronous manner [7].

In this proposal we focus on the design of synchronizing controllers for an ensemble of neurons, which are applied to a small number of neurons, in [8], the synchronization of small-world neuron networks is presented, however, the synchronization is carried out by means of the synapses between the neurons. Some results present the synchronization between neurons in a network [9], [10], where the synchronization is studied as a collective activity obtained by varying certain parameters in the neuron model, the incoming current and the coupling strength. In this contribution the objective is to impose the spiking-bursting activity on the ensemble of neurons which are in their resting state by means of a controller which provides the advantage of imposing any other neuron behavior. The active state of neurons is characterized by multiple spikes, which is caused by the interactions of slow and a fast processes. These features of the active state in neurons are well documented in biology, for example the neurons in the thalamus in spike-burst activity produce periods of drowsiness, inattentiveness or sleep [11]; another example of oscillations produced by spike-burst synchrony in the neurons is the slow oscillatory behavior of neurons that naturally occurs in sleeping mammals [11–16].

In this contribution, we focused on the imposition of a particular pattern to an ensemble of a class of neurons described by the Hindmarsh-Rose (HR) model [17] in a given topology. The activation procedure consists in to control only a reduced number of neurons in the network, thus the synchronization is accomplished taking the advantage of the collectivity of the network and it is not re-

Gualberto Solís-Perales is with Departamento de Electrónica, CUCEI-Universidad de Guadalajara, Av. Revolución No. 1500 Guadalajara Jalisco, C.P. 44430, México. Phone: (+52) 33 1378-5900 (e-mail: gualberto.solis@cucei.udg.mx).

Juan Gonzalo Barajas-Ramírez is with IPICyT, División de Matemáticas Aplicadas, Camino a la Presa San José 2055, Col. Lomas 4a Sección, San Luis Potosí, S.L.P., C.P. 78216, México. Phone: (+52) 444 834-2000 (e-mail: jgbarajas@ipicyt.edu.mx).

quired a controller for each neuron. Some published results deal with the synchronization of neuronal networks considering as many controllers than the number of neurons in the network, moreover, they synchronize two equal networks with the same topology by means of controlling each neuron in the response network [18], [19], [20]. In our proposal we assume that the neurons in the network are in a silent state, with the end of illustrate the activation, but can be considered other behavior of the neurons. Then, applying the synchronizing controller to some neurons, a desired pattern of activation can be imposed in the entire network. We show that a simple classical controller can practically impose a pattern of activation on a ensemble of HR neurons.

The rest of the manuscript is organized as follows: Section 2 describes the problem of the activation of a network of neurons; Section 3 presents the procedure to impose the activation pattern on the neural ensemble via the controlled synchronization; in Section 4 numerical simulations are presented to illustrate the activation of the network and finally, Section 5 closes the contribution with some concluding comments.

2. PROBLEM DESCRIPTION

2.1. Model of a Single Neuron

Each node in the network is a neuron and is described by the $4D$ HR model [21], which is an extension of the $3D$ model reported by Hindmarsh and Rose [17]

$$\begin{aligned} \dot{x} &= y - ax^3 - bx^2 + I - z \\ \dot{y} &= c - dx^2 - y \\ \dot{z} &= r(s(x - x_r) - z) \end{aligned} \tag{1}$$

The $4D$ model is considered due to its facility to be implemented electronically and all its potential applications in syntectic biology and the model is given by the set of equations

$$\begin{aligned} \dot{x}_{i1} &= \alpha x_{i2} + \beta x_{i1}^2 - \gamma x_{i1}^3 - \delta x_{i3} + I_i \\ \dot{x}_{i2} &= \varepsilon - \sigma x_{i1}^2 - x_{i2} - \zeta x_{i4} \\ \dot{x}_{i3} &= \eta(-x_{i3} + S(x_{i1} + h)) \\ \dot{x}_{i4} &= \theta(-\vartheta x_{i4} + \iota\,(x_{i2} + \kappa)) \\ y_i &= \Gamma x_i \end{aligned} \tag{2}$$

where $x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4})^\top \in \mathbb{D} \subset \mathbb{R}^4$ is the vector states of the i-th neuron and $\mathbb{D}$ is the subspace for all admissible x_i, parameters α,β, γ, δ, ε, σ, ζ, η, θ, ϑ, ι and κ, are constant parameters which embody the underlying current and conductance dynamics in this polynomial representation of the neural dynamics, $\Gamma = diag[1,0,0,0]$ is the output matrix and indicates which state in the system is the output, for neuron systems the output is the current produced by the membrane potential (electrical synapse) represented by x_{i1}; x_{i2} represents the "fast" current, while x_{i3} represents the "slow" current of the ion dynamics (with $\eta \ll 1$); and x_{i4} represents an even slower current dynamics (with $\theta < \eta \ll 1$) which models the calcium exchange between the intracellular stores and the cytoplasm. As it was mentioned, the main idea is to impose a particular pattern in the network using a classical controller, therefore, we are interested in reactivating the neurons in the network or induce the spiking-bursting activity in the membrane potential as illustrated in Fig. 1.

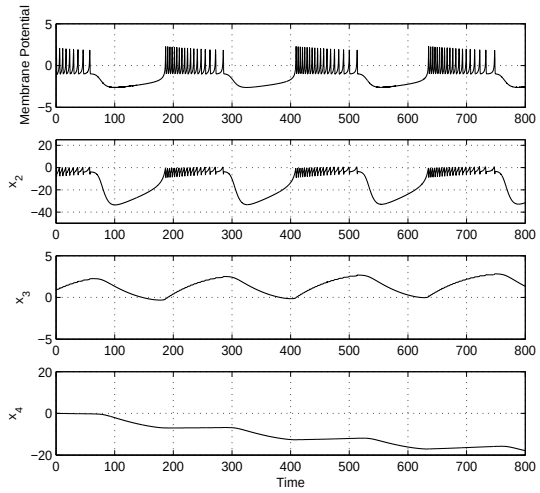

Fig 1: Spiking-bursting activity of the HR neuron (reference), this is the desired activation pattern.

On the other hand, in order to obtain a resting state in the behavior of the HR neuron, the parameter α is varied, thus the influence of the fast current in the neuron is diminished and the spike-burst regime is reduced or suppressed and then the operation of the neurons become a fixed point as presented in Fig. 2. In this way some of the neurons in the network reproduce a silent behavior and then the collective behavior is an equilibrium point. Therefore, the main problem is to activate the neurons in the network by means of a controller which imposes a particular behavior in the controlled neurons.

2.2. Ensemble Description

We consider a group of neurons described by (2) coupled via their electrical activity represented by the membrane potential x_{i1}. In the reminder of this contribution we will consider that this is the unique form of communication between neurons in the ensemble. Therefore, the i-th neuron transmits the current x_{i1} to the j-th neuron through the connections of the network, this current is algebraically added to the currents coming from all other neurons to form the incoming external electrical activity. Then, the state space description of a network with N lin-

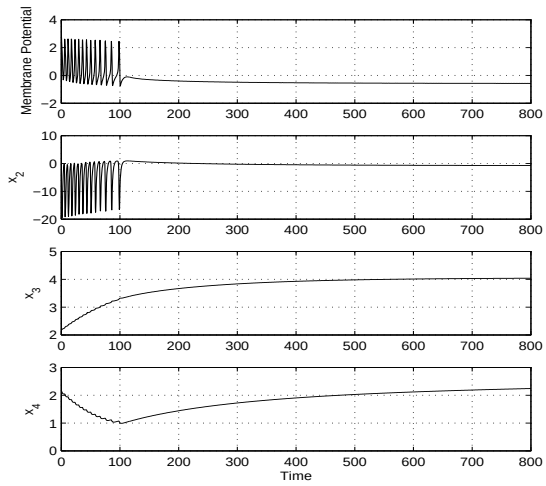

Fig 2: HR model in its resting state, it is stabilized at an equilibrium.

early and diffusively coupled HR neurons is given by:

$$\dot{x}_i = F(x_i) + c\sum_{j=1}^{N} \mathscr{L}_{ij}\Gamma x_j \tag{3}$$

where x_i are the state vector of the i-th neuron on the network; $F : \mathbb{D} \to \mathbb{R}$ is a smooth vector field representing the dynamics of the i-th neuron. The inner coupling matrix is of the form $\Gamma = diag[1,0,0,0]$ and the connections describing the topology are given by the Laplacian matrix $L = \{\mathscr{L}_{ij}\} \in \mathbb{R}^{N\times N}$, if there is connection (coupling) between the i-th and j-th neurons then $\mathscr{L}_{ij} = \mathscr{L}_{ji} = 1$ for $i \neq j$ otherwise $\mathscr{L}_{ij} = \mathscr{L}_{ji} = 0$ and the diagonal elements satisfies $\mathscr{L}_{ii} = -\sum_{j=1,j\neq i}^{N} \mathscr{L}_{ij}$

As stated in the introduction, the imposition of an activation pattern in the inhibited ensemble is desired. The proposed ensemble is illustrated in Fig. 3, note that there are some nodes highly connected whereas many nodes only have one connection [22]. For the imposition of the activation pattern we propose to control nodes 1, 3 and 14. To this end, in the following section a controller is proposed to force the entire network to follow a reference membrane potential behavior ($x_{Ref,1}$).

3. IMPOSING AN ACTIVATION PATTERN VIA CONTROLLED SYNCHRONIZATION

The trivial case for imposing a pattern in a network is when every system is controlled or forced to track a prescribed reference [18], [19], [20], therefore, the main idea is to control $\rho < N$ nodes in the network. This means that controlling a small number of nodes, the synchronous behavior is induced or imposed to the whole network. The network description including the control input is given as

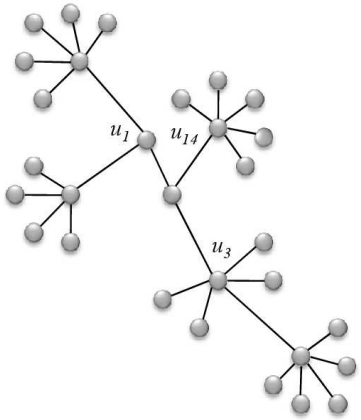

Fig 3: Network topology considered for the ensemble of HR, where u_1, u_3 and u_{14} are controllers applied in the nodes 1, 3 and 14. neurons.

follows

$$\begin{aligned} \dot{x}_{Ref}(t) &= Ax_{Ref}(t) + f(x_{Ref}(t)) \\ \dot{x}_i(t) &= Ax_i(t) + f(x_i(t)) + c\sum_{j=1}^{N} \mathscr{L}_{i,j}\Gamma x_j(t) + \mu_i \end{aligned} \tag{4}$$

where A is the matrix for the linear part of the neuron system, $x_{Ref}(t)$ is the state vector of the reference system, which is not influenced by the rest of the systems, μ_i is the input vector control to the i-th system, $f(x_i(t))$ is the nonlinear vector field lumping the nonlinearities of the systems and satisfies $\|f(x_i) - f(x_j)\| \leq \phi_i\|x_i - x_j\|$, for some positive ϕ_i, $\mathscr{L}_{i,j}$ are the entries of the Laplacian matrix L which describe the network topology. This control feedback law is given as $\mu_i = \tau_i K_i e_i(t)$ where $e_i \in \mathbb{D}$ $e_i(t) = x_{ref}(t) - x_i(t)$ describes the synchronization error between the reference and the i-th system, $\tau_i = 1$ corresponds to the case of a controlled system and $\tau_i = 0$ if the system is not controlled and $K \in \mathbb{R}^{n\times n}$ is the gain matrix, $K_i = diag[k_i,0,0,0]$. The controlled systems can be determined by setting zero or one the elements of the diagonal matrix $T = diag[\tau_1,\tau_2,\cdots,\tau_N]$. With this information and using the Kronecker product one can construct a nN dimensional system considering $\chi(t) = [x_1(t)^T, x_2(t)^T, \cdots, x_N(t)^T]^T$

$$\begin{aligned} \dot{\chi}(t) = I_{N\times N} \otimes A\chi(t) + \mathscr{F}(\chi(t)) + cL \otimes \Gamma\chi(t) + \\ T \otimes K(X_{Ref}(t) - \chi(t)) \end{aligned} \tag{5}$$

where $\mathscr{F}(\chi(t)) = [f(x_1(t))^T, f(x_2(t))^T, \cdots, f(x_N(t))^T]^T$ is a nN vector lumping the vector fields of the neurons and $X_{Ref}(t) = [x_{Ref}(t)^T, x_{Ref}(t)^T, \cdots, x_{Ref}(t)^T]^T$. The designing parameters are the coupling strength c and the feedback gain K such that the network synchronizes to the reference $x_{Ref}(t)$. The synchronization problem can be viewed as a stabilization of a dynamical error system between the reference and the controlled systems in the network, thus, from (5) the dynamical error system is as fol-

lows

$$\dot{e}(t) = I_{N\times N}\otimes Ae(t) + \mathscr{F}(X_{Ref}) - \mathscr{F}(\chi(t)) + cL\otimes\Gamma e(t) - T\otimes Ke(t) \quad (6)$$

now it can be defined a matrix function $\Psi_{x,\bar{x}}$ for any $x,\bar{x}\in\mathbb{D}$ by

$$\Psi_{x,\bar{x}}(x-\bar{x}) = f(x) - f(\bar{x}) \quad (7)$$

where the matrix $\Psi_{x,\bar{x}}\in\mathbb{D}$ is bounded and there exists a positive constant α such that $\|\Psi_{x,\bar{x}}\|\leq\alpha$ for any $x,\bar{x}\in\mathbb{D}$. Condition (7) comes from the Lipschitz condition and the mean value theorem [23], [24]. Therefore from (6) and (7) the synchronization error system for the controlled network is written as follows

$$\dot{e}(t) = I_{N\times N}\otimes Ae(t) + \Phi(e(t))e(t) + (cL\otimes\Gamma - T\otimes K)e(t) \quad (8)$$

where $\Phi(e(t)) = Diag[\Psi_{x_{Ref},x_{Ref}-e_1(t)},\cdots,\Psi_{x_{Ref},x_{Ref}-e_N(t)}]$, thus the synchronization between systems is achieved if system (8) is asymptotically stable at the origin. Note that the stability of system (8) depends on the value of the control gains of the feedback control law and the coupling strength of the network. In the following Lemma a condition to achieve synchronization is provided, however, the choice of the controlled nodes and the determination of ρ is not studied.

Lemma 1. If there exist control gains K, a coupling strength c and given an appropriate matrix Γ, for every $e_i\in\mathbb{D}$, $\lambda_j(e) < -\sigma$ with $j = 1,2,\cdots,nN$, $\sigma > 0$, where $\lambda_j(e)$ is the j-th eigenvalue of the matrix

$$(I_N\otimes P)(I_{N\times N}\otimes A + \Phi(e(t)) + (cL\otimes\Gamma - T\otimes K)) + (I_{N\times N}\otimes A + \Phi(e(t)) + (cL\otimes\Gamma - T\otimes K))^T(I_N\otimes P)$$

with $P\in\mathbb{R}^{n\times n}$ some positive definite matrix, then the synchronization error system (8) is asymptotically stable at the origin, which implies that the systems in the network (4) are asymptotically synchronized.

Poof. Choose the candidate Lyapunov function

$$V(e) = e^T(I_N\otimes P)e$$

Thus its time derivative is

$$\begin{aligned}\dot{V}(e) &= e^T(I_N\otimes P)\dot{e} + \dot{e}^T(I_N\otimes P)e\\ &= e^T(I_N\otimes P)(I_{N\times N}\otimes A + \Phi(e(t)) + (cL\otimes\Gamma - T\otimes K))e + e^T(I_{N\times N}\otimes A + \Phi(e(t)) + (cL\otimes\Gamma - T\otimes K))^T(I_N\otimes P)e\end{aligned}$$

now considering the boundedness of $\Phi(e(t))$ we have

$$\begin{aligned}\dot{V}(e) &\leq e^T(I_N\otimes P)(I_{N\times N}\otimes A + \bar{\Psi} + (cL\otimes\Gamma - T\otimes K))e + e^T(I_{N\times N}\otimes A + \bar{\Psi} + (cL\otimes\Gamma - T\otimes K))^T(I_N\otimes P)e\end{aligned}$$

where $\bar{\Psi} = Diag[\alpha_1,\alpha_2,\cdots,\alpha_N]$, now let us define $Q = I_{N\times N}\otimes A + \bar{\Psi} + (cL\otimes\Gamma - T\otimes K)$ thus the derivative can be writing as

$$\begin{aligned}\dot{V}(e) &\leq e^T(I_N\otimes P)Qe + e^TQ^T(I_N\otimes P)e\\ &\leq e^T\{(I_N\otimes P)Q + Q^T(I_N\otimes P)\}e\end{aligned}$$

let $R = (I_N\otimes P)Q + Q^T(I_N\otimes P)$ and since $R = R^T$ thus $R = U^*\Lambda U$, where U is a square unitary matrix and $\Lambda = Diag(\lambda_1,\lambda_2,\cdots,\lambda_{nN})$. Then

$$\dot{V}(e)\leq e^TRe\leq e^TU^*\Lambda Ue\leq\tilde{e}^T\Lambda\tilde{e}\leq\sigma\|\tilde{e}\| < 0$$

where $\tilde{e} = Ue$, therefore, according to the Lyapunov stability theory system (8) is asymptotically stable about the origin, hence the network is synchronous and this complete the proof.

The previous Lemma provides conditions for the controlled synchronization of networks, and for the case of networks of neurons provides conditions for the imposition of a particular behavior in the whole network. Moreover, the synchronization between neurons is achieved even if the neurons behavior is not in the resting state, since the input in each neuron has the information required to stimulate the neuron such that it follows its neighbors.

4. SIMULATION RESULTS

With the previous results we consider the network arrangement illustrated in Fig. 3 with $N = 30$ and with three controlled nodes. The parameters for the neurons in the nodes are $\beta = 3$; $\gamma = 1$; $\delta = 0.99$; $I = 0$; $\varepsilon = 1.01$; $\sigma = 5.0128$; $\zeta = 0.0278$; $\eta = 0.0021$; $S = 3.966$; $h = 1.605$; $\theta = 0.0009$; $\vartheta = 0.9573$; $\iota = 3.0$; $\kappa = 1.619$. The parameter $\alpha = (1+\Delta)\alpha_0$ with $0 < \Delta < 0.1$ and $\alpha_0 = 1$, produces the resting behavior in some neurons. The synchronization error system is given by

$$\begin{aligned}\dot{e}_{i1} &= \alpha e_{i2} - \delta e_3 + (\beta x_{Ref,1}^2 - \gamma x_{Ref,1}^3) - (\beta x_{i1}^2 - \gamma x_{i1}^3) - \tau_i u\\ \dot{e}_{i2} &= -e_{i2} - \zeta e_{i4} + (-\sigma x_{Ref,1}^2 + \sigma x_{i1}^2)\\ \dot{e}_{i3} &= -\eta e_{i3} + \eta S e_{i1}\\ \dot{e}_{i4} &= -\theta\vartheta e_{i4} + \theta\iota e_{i2}\end{aligned} \quad (9)$$

From this system we proceed to determine the largest eigenvalue of the matrix $R = (I_N\otimes P)Q + Q^T(I_N\otimes P)$, for values of $c\in[1000,5000]$ and for the gain matrix $K = diag[k_1,0,0,0]$ with $k_1\in[1000,3500]$ is applied to the nodes 1, 3 and 14, as illustrated in Fig. 3. Then with this set up, the control gains and the coupling strength are determined numerically following Lemma 1, thus the largest eigenvalues for several values of the control gains and the couplings are illustrated in Fig. 4. From this graph the largest eigenvalue is $\lambda_{max} = -0.0012$ for the control gains $K_1 = K_2 = K_3 = 3500$ and $c = 5000$ and then there exists a constant $-\sigma > \lambda_{max}$, such that the synchronization error is asymptotically stabilized at the origin. Once we have the control

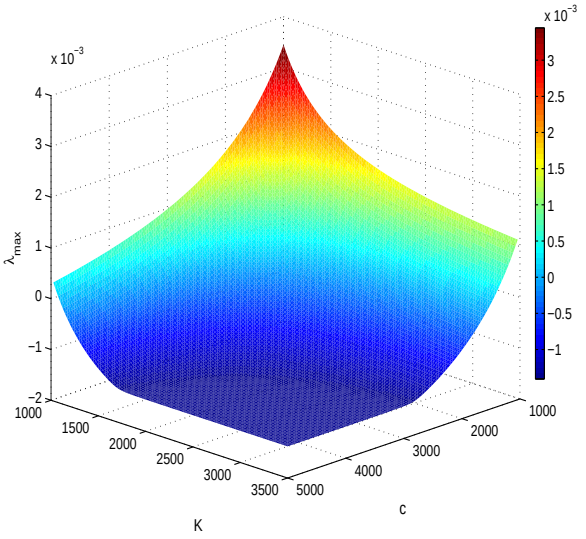

Fig 4: Largest eigenvalue λ_{max} of the matrix R which defines the stability of the closed-loop system, the color bar represents the intensity of the largest eigenvalue.

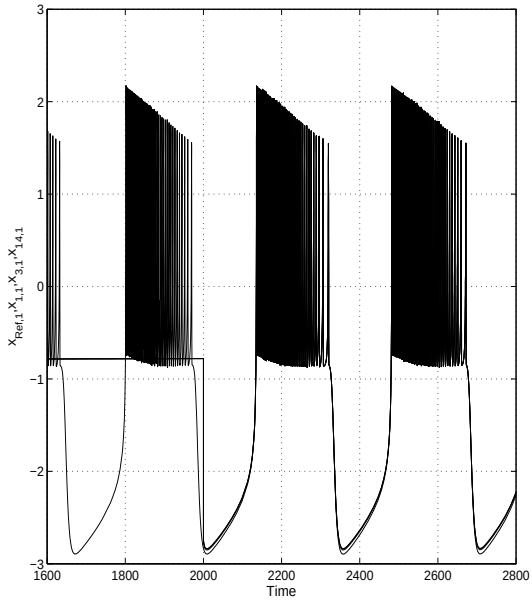

Fig 5: Synchronization behavior of the controlled neurons, $x_1,1$, $x_1,3$ and $x_1,14$ are the membrane potentials of the neurons which synchronize with the reference potential.

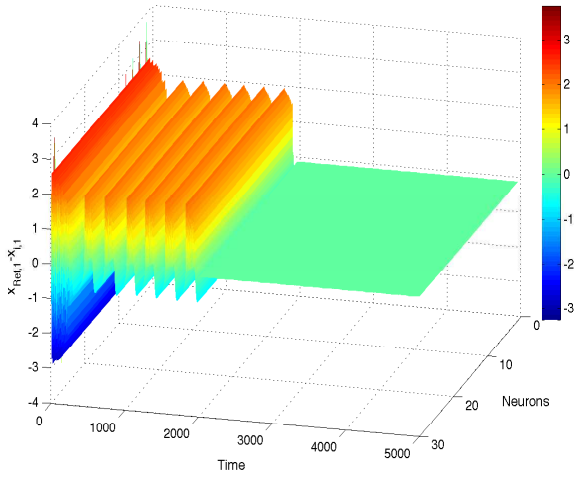

Fig 6: Synchronization error behavior of each neuron in the network, since the error remains close to zero, the synchronization in all neurons is achieved, the color bar represents the error value.

gains and the coupling strength, we consider that the ensemble is synchronous in an inhibited behavior, in other words, the ensemble is uncontrolled. The idea is to induce the desired behavior in the ensemble by controlling three neurons. Therefore, the controllers are connected at time $t \geq 2000$. Clearly, the ensemble is reactivated, and the spike-burst behavior is present in the whole ensemble with control gains $K_1 = K_3 = K_{14} = 3500$ and the coupling strength $c = 5000$ as illustrated in Fig. 5, where after the connection of the controllers the behavior of the neurons in the whole network follows the signal reference $x_{Ref,1}$. In Fig. 6 the synchronization errors for each neuron is illustrated, this indicates that the potential in each neuron in the network follows the reference signal $x_{Ref,1}$. It is observed that the controllers activate the behavior in the network, and for the effect of the coupling between nodes the network synchronizes to the reference neuron. There are two aspects that should be mentioned, the first concerns with the control gains such that the controlled neuron is forced to track the reference signal in such a way that the controlled neuron behavior is propagated to uncontrolled neurons in the network. The second aspect is related to the coupling and the topology of the network, in this case the coupling strength between neurons is difficult to manipulate therefor, the synchronization can be obtained via the control gains, in the same way the structure of the network is uncertain but it is considered a certain static structure to illustrate the activation scheme. Compared with existing results, the synchronization is achieved using a reduce number of classical controllers, whereas some of the results use many controllers as the number of nodes in the network [18], [19], [20]. On the other hand, a similar

result uses less controllers than the number of nodes, however use an adaptive control scheme is proposed [25], the adaptive controllers increase the order of the system and the computational cost, since it is required to dynamically adapt the parameters of the control system.

5. CONCLUSION

In this contribution we present the induction of a desired behavior into a network of inhibited neurons. The main idea was to control some nodes in the network through which the synchronous behavior is propagated to all the neurons in the network. The propagation effect is produced by the interconnection of the nodes in the network and due to the coupling strength. The controller used in this contribution was a classical proportional gain, however, some other control technique can be applied to obtain a better performance. The main contribution consists in general terms in the activation or re-synchronized of a network of neurons in a different synchronization manifold which can be established a priori, this means that a neuronal ensemble can be synchronized with any behavior dictated by any other artificial or biological neuron [21]; this can be done via controlling some nodes in the network and such a controller and the reference neuron in practical terms could be implemented in an electronic circuit. The result is somewhat conservative since the controller performance for tracking is very limited, a future contribution is to design another class of controllers. On the other hand, some challenging tasks remain, for instance, the number of controlled nodes, which nodes in the network are to be controlled, how to control an evolutive network, etc.

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Gualberto Solís-Perales Received the B.S. degree in Electronic Engineering from the Faculty of Science from the Autonomous University of San Luis Potosí, México in 1995, the M.Sc. and PhD. degree in Electrical Engineering from the Autonomous University of San Luis Potosí 1999, 2002 respectively. His research interests include nonlinear control theory and its application to control of complex systems and synchronization of chaos. More precisely the control and synchronization of complex networks and its application.

Juan Gonzalo Barajas-Ramírez Received the B.S. degree in Industrial Electronic Engineering from the Tijuana Technological Institute, México in 1995, the M.Sc. degree in Electronics and Telecommunications from the CICESE 1997 and the PhD degree in Electrical Engineering from the University of Houston in 2002. His research interests include nonlinear dynamics and chaos, complex systems and synchronization of chaos, dynamics of complex networks and its application.