FRENET-SERRET VACUUM RADIATION, DETECTION PROPOSALS AND RELATED TOPICS

Haret C. Rosu[1]‡

[1] Dept. of Applied Mathematics
Potosinian Institute of Scientific and Technological Research
Apdo Postal 3-74 Tangamanga, San Luis Potosí, MEXICO

Dated: January 2003

Abstract

The paradigmatic Unruh radiation is an ideal and simple case of stationary vacuum radiation patterns related to worldlines defined as Frenet-Serret curves. We review the corresponding body of literature as well as the experimental proposals that have been suggested to detect these types of quantum field radiation patterns. Finally, we comment on a few other topics related to the Unruh effect.

extended version of invited talk at QABP3, Hiroshima, Japan, January 7-11, 2003

‡ hcr@ipicyt.edu.mx

1. FRENET-SERRET VACUUM RADIATION

J.F. Frenet (1816-1900) wrote a doctoral thesis in 1847. Part of it deals with the theory of space curves and contains six formulas of the nine three-dimensional Frenet-Serret formulas. He published his results in *Journal de mathematiques pures et appliques* in 1852. J.A. Serret (1819-1885) gave the set of all nine formulas in three dimensions.

The standard FS formulas are the following

$$\dot{\vec{T}} = \kappa \cdot \vec{N}$$
$$\dot{\vec{N}} = \tau \cdot \vec{B} - \kappa \cdot \vec{T}$$
$$\dot{\vec{B}} = - \tau \cdot \vec{N} \ .$$

These three formulas give the derivatives of the unit tangent, normal, and binormal vectors, respectively, as vectorial combinations of the moving basis in which the two Frenet-Serret invariants of the normal three-dimensional space, curvature and torsion, respectively, enter as scalar coefficients. They can be written in matrix form as well

$$\begin{pmatrix} \dot{\vec{T}} \\ \dot{\vec{N}} \\ \dot{\vec{B}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}$$

The plane formed by the span of $\vec{T}$ and $\vec{N}$ is called the osculating plane. The span of $\vec{N}$ and $\vec{B}$ is the normal plane and the span of $\vec{B}$ and $\vec{T}$ is the transverse plane to the curve at the point where the Frenet-Serret triad is considered.

For the goals of the first part of this review, the interested reader may consult several papers of more recent times. These are seminal papers that apply the Frenet-Serret formalism to relativistic world lines and quantum field theory. First, as quoted by J.L. Synge, Jasper [1] studied in 1947 helices in flat space of *n* dimensions, but only for a positive-definite metric. Next, Synge wrote an important paper on timelike helices in flat space-time and classified them in six types according to the relative magnitudes of the three Frenet-Serret invariants [2]. He also proved for the first time that the world line of a charged particle moving in a constant electromagnetic field is a helix. A few years later, in the seminal paper of Honig, Schucking and Vishveshwara [3] a connection was established between the Lorentz invariants of the elctromagnetic field and the Frenet-Serret invariants. The 1981 Letaw's paper [4] is in fact the first work in which the FS invariants and quantum field concepts are merged together (see below). Interesting generalizations to more dimensions, black hole environments, and gyroscopic precession have been investigated by Iyer and Vishveshwara [5] (see also [6]).

We also notice that at the strictly classical level, the Frenet-Serret coordinate system is quite used in accelerator physics to express Hamiltonians and/or fields [7].

1.1. **Stationary world lines and the vacuum excitation of noninertial detectors**

In 1981, Letaw studied the stationary world lines, on which quantized field detectors in a scalar vacuum have time-independent excitation spectra. They are characterized by the requirement that the geodetic interval between two points depends only on the proper time interval. He used a generalization of the Frenet equations to Minkowski space and found, not surprisingly, that the curvature invariants are the proper acceleration and angular velocity of the world line. Solving the generalized Frenet equations for the simple case of constant invariants leads to several classes of stationary world lines. Letaw gave a classification into six types reviewed here. He also demonstrated the equivalence of the timelike Killing vector field orbits and the stationary world lines. Last but not least, Letaw did some calculations of the vacuum excitation spectra of detectors on the sample of six families of stationary world lines. Letaw's work is a generalization of Unruh's famous result concerning the excitation of a scalar-particle detector moving with constant linear acceleration in the vacuum of flat spacetime. According to Unruh, the detector behaves as if in contact with a bath of scalar particles with energies in a Planck spectrum of temperature: acceleration/2π. Because of the connection with the Hawking radiation and its paradigmatic nature, the Unruh effect attracted the attention of many theoretical physicists. On the other hand, the present author considers that Letaw's results place the Unruh effect in a different and more general perspective that is elaborated herein. We hope that a more effective attitude is put forward in the present review that could be more rewarding in the years to come and may help to strengthen the links between laboratory physics and astrophysics.

According to DeWitt, the probability for a detector moving along a world line $x^{\mu}(\tau)$ to be found in an excited state of energy E at $\tau = \tau_0$ is given in terms of the autocorrelation function of the field (Wightman function)

$$P(E) = D(E) \int_{-\infty}^{\tau_0} d\tau \int_{-\infty}^{\tau_0} d\tau' e^{-iE(\tau - \tau')} \langle 0|\phi(x(\tau))\phi(x(\tau'))|0 \rangle , \qquad (1)$$

where $D(E)$ is a function characterizing the sensitivity of the detector. The Wightman function for a scalar field is proportional to the inverse of the geodetic interval $\Delta(\tau, \tau')$

$$\langle 0|\phi(x(\tau))\phi(x(\tau'))|0 \rangle = [2\pi^2 \Delta(\tau, \tau')]^{-1} , \qquad (2)$$

where

$$\Delta(\tau, \tau') = [x_{\mu}(\tau) - x_{\mu}(\tau')][x^{\mu}(\tau) - x^{\mu}(\tau')] . \qquad (3)$$

The rate of excitation to the state with energy E is the Fourier cosine transform

$$\frac{dP(E)}{d\tau_0} = 2D(E) \int_{-\infty}^{0} ds \langle 0|\phi(x(\tau_0))\phi(x(\tau_0 + s))|0 \rangle \cos(Es) . \qquad (4)$$

where $s = \tau - \tau'$ is the proper time interval. The rate of excitation (the response) is directly related to the energy spectrum of the detected scalar "particles"

$$S(E, \tau) = 2\pi\rho(E) \int_{-\infty}^{0} ds \langle 0|\phi(x(\tau))\phi(x(\tau + s))|0 \rangle \cos(Es) . \qquad (5)$$

If the Wightman function is time independent the detected spectra are stationary. This is equivalent to the following property of the geodetic interval

$$\Delta(\tau, \tau + s) = \Delta(0, s) . \qquad (6)$$

Curvature invariants and the Frenet equations

An arbitrary timelike world line in flat space is generally described by four functions, $x^\mu(s)$, specifying the coordinates of each point s on the curve. This parameter may be taken to be the arc length or proper time on the world line. The parametric representation is unsatisfactory in two respects:

(1) *A world line is a geometric object and should not require a coordinate-dependent entity for its definition.*

(2) *There is an inherent redundancy in the parametric representation since three functions suffice to determine the world line.*

The Frenet-Serret (curvature) invariants, on the other hand, provide an intrinsic definition of the world line not subject to these criticisms.

The starting point is the construction of an orthonormal tetrad $V_a^\mu(s)$ at every point on the world line $x^\mu(s)$. The Latin index everywhere is a tetrad index. The tetrad is formed from the derivatives of $x^\mu(s)$ with respect to proper time (represented by one or more dots). It is assumed that the first four derivatives are linearly independent, the results being practically unchanged when they are not. Members of the tetrad must satisfy the orthonormality condition

$$V_{a\mu}V_b^\mu = \eta_{ab} , \tag{7}$$

where the metric has diagonal components (1,-1,-1,-1) only.

By Gram-Schmidt orthogonalization of the derivatives working upwords from the first, the following expressions for the tetrad are found:

$$V_0^\mu = \dot{x}^\mu , \tag{8}$$

$$V_1^\mu = \frac{\ddot{x}^\alpha}{(-\ddot{x}_\alpha \ddot{x}^\alpha)^{1/2}} , \tag{9}$$

$$V_2^\mu = \frac{(\ddot{x}_\gamma \dddot{x}^\gamma)\,\ddot{x}^\mu - (\ddot{x}_\gamma\,\dddot{x}^\gamma)\ddot{x}^\mu + (\ddot{x}_\gamma\dddot{x}^\gamma)^2\dot{x}^\mu}{[(\ddot{x}_\alpha\ddot{x}^\alpha)^4 + (\ddot{x}_\alpha\ddot{x}^\alpha)(\ddot{x}_\beta\,\dddot{x}^\beta)^2 - (\ddot{x}_\alpha\ddot{x}^\alpha)^2(\dddot{x}^\beta\dddot{x}^\beta)]^{1/2}} , \tag{10}$$

$$V_3^\mu = \frac{1}{\sqrt{6}}\varepsilon^{\alpha\beta\gamma\mu}V_{0\alpha}V_{1\beta}V_{2\gamma} , \tag{11}$$

Overall signs on these vectors are fixed by the orientation of the tetrad.

The tetrad V_σ^μ is a basis for the vector space at a point on the world line. Derivatives of the basis vectors may therefore be expanded in terms of them:

$$\dot{V}_a^\mu = K_a^b V_b^\mu . \tag{12}$$

These are the generalized Frenet equations. K_{ab} is a coordinate-independent matrix whose structure must be determined.

Differentiation of the orthonormality condition (7) yields

$$\dot{V}_{a\mu}V_b^\mu + V_{a\mu}\dot{V}_b^\mu = 0 , \tag{13}$$

and, in view of (12),

$$K_{ab} = -K_{ba} . \tag{14}$$

A basis vector V_a^μ is defined in terms of the first $a+1$ derivatives of x^μ; therefore, $\dot{V}_a^\mu$ will be a linear combination of the first $a+2$ derivatives. These $a+2$ derivatives are dependent only on the basis vectors V_b^μ where $b \leq a+1$. This and (14) limit the matrix to the form

$$K_{ab} = \begin{pmatrix} 0 & -\kappa(s) & 0 & 0 \\ \kappa(s) & 0 & -\tau_1(s) & 0 \\ 0 & \tau_1 & 0 & -\tau_2 \\ 0 & 0 & \tau_2 & 0 \end{pmatrix} \tag{15}$$

The three functions of proper time are the invariants

$$\kappa = V_{0\mu}\dot{V}_1^\mu = -\dot{V}_{0\mu}V_1^\mu \ , \tag{16}$$

$$\tau_1 = V_{1\mu}\dot{V}_2^\mu = -\dot{V}_{1\mu}V_2^\mu \ , \tag{17}$$

$$\tau_2 = V_{2\mu}\dot{V}_3^\mu = -\dot{V}_{2\mu}V_3^\mu \ . \tag{18}$$

They are, respectively, the curvature, the first torsion, and the second torsion (hypertorsion) of the world line. Sign choices are made for reasons brought out below.

To explore the physical significance of the invariants we examine the infinitesimal Lorentz transformations of the tetrad at a point on the world line. The transformations leave the metric invariant

$$\eta_{ab} = L_a^c L_b^d \eta_{cd} \ . \tag{19}$$

An infinitesimal transformations may be written

$$L_a^c = \delta_a^c + d\varepsilon_a^c \ , \tag{20}$$

where the elements of $d\varepsilon_a^c$ are small and must satisfy

$$d\varepsilon_{ab} = -d\varepsilon_{ba} \ , \tag{21}$$

The transformations are taken to be active; that is, the transformed tetrad moves $+v$ and is rotated θ relative to the untransformed tetrad. Thus the infinitesimal generator is

$$d\varepsilon_{ab} = \begin{pmatrix} 0 & -dv_1 & -dv_2 & -dv_3 \\ dv_1 & 0 & -d\theta_{12} & d\theta_{31} \\ dv_2 & d\theta_{12} & 0 & -d\theta_{23} \\ dv_3 & -d\theta_{31} & d\theta_{23} & 0 \end{pmatrix} \tag{22}$$

The change in the tetrad resulting from this transformation is

$$\dot{V}_a^\mu = (d\varepsilon_a^b/ds)V_b^\mu \ . \tag{23}$$

Equations (23) are identical to the Frenet equations (12); therefore, the physical content of the curvature invariants is found by comparison of (15) and (22).

Physical interpretation of the FS invariants

1. κ is the proper acceleration of the world line which is always parallel to V_1^μ.

2. τ_1 and τ_2 are the components of proper angular velocity of the world line in the planes spanned by V_1^μ and V_2^μ, and V_2^μ and V_3^μ, respectively. The total proper angular velocity is the vector sum of these two invariants.

Stationary Motions

The simplest worldlines are those whose curvature invariants are constant. They are called *stationary* because their geometric properties are independent of proper time. One also finds that only observers on these world lines may establish a coordinate system in which they are at rest and the metric is stationary. Clearly, the geodetic interval between two points on a stationary world line can depend only on the proper time interval, therefore they are the world lines on which a detector's excitation is time independent.

The Frenet equation (12) may be reduced to a fourth-order linear equation in V_0^μ when the curvature invariants are constant

$$\ddddot{V}_0^\mu - 2a\ddot{V}_0^\mu - b^2 V_0^\mu = 0 , \tag{24}$$

where

$$a = \frac{1}{2}\left(\kappa^2 - \tau_1^2 - \tau_2^2\right), \quad b = |\kappa\tau_2| .$$

The other basis vectors are determined from V_0^μ by the equations

$$V_1^\mu = \dot{V}_0^\mu/\kappa \tag{25}$$

$$V_2^\mu = (\ddot{V}_0^\mu - \kappa^2 V_0^\mu)/\kappa\tau_1 \tag{26}$$

$$V_3^\mu = [\dddot{V}_0^\mu - (\kappa^2 - \tau_1^2)\dot{V}_0^\mu]/\kappa\tau_1\tau_2 . \tag{27}$$

Equation (24) is homogeneous with constant coefficients. The four roots of the characteristic equation are $\pm R_+$ and $\pm iR_-$, where

$$R_\pm = [(a^2 + b^2)^{1/2} \pm a]^{1/2} . \tag{28}$$

The solution is of the form

$$V_0^\mu = A^\mu\cosh(R_+ s) + B^\mu\sinh(R_+ s) + C^\mu\cos(R_- s) + D^\mu\sin(R_- s) . \tag{29}$$

Using (25)-(27) and (29) at $s = 0$ and the initial conditions $(V_a^\mu)_{s=0} = \delta_a^\mu$ one can get the following expressions for the coefficients

$$A^\mu = R^{-2}(R_-^2 + \kappa^2, 0, \kappa\tau_1, 0) , \tag{30}$$
$$B^\mu = R^{-2}(0, \kappa(R_-^2 + \kappa^2 - \tau^2)/R_+, 0, \kappa\tau_1\tau_2/R_+) , \tag{31}$$
$$C^\mu = R^{-2}(R_+^2 - \kappa^2, 0, -\kappa\tau_1, 0) , \tag{32}$$
$$D^\mu = R^{-2}(0, \kappa(R_+^2 - \kappa^2 + \tau^2)/R_-, 0, -\kappa\tau_1\tau_2/R_-) , \tag{33}$$
$$\tag{34}$$

with $R^2 = R_+^2 + R_-^2$. From these results on the FS tetrad one gets easily the six classes of stationary world lines and the corresponding excitation spectra.

The Six Stationary Scalar Frenet-Serret Radiation Spectra

1. $\kappa = \tau_1 = \tau_2 = 0$ (inertial, uncurved worldlines)

The excitation spectrum is a trivial cubic spectrum

$$S_{\text{recta}}(E) = \frac{E^3}{4\pi^2} , \tag{35}$$

i.e., as given by a vacuum of zero point energy per mode $E/2$ and density of states $E^2/2\pi^2$.

2. $\kappa \neq 0$, $\tau_1 = \tau_2 = 0$ (hyperbolic worldlines)

The excitation spectrum is Planckian allowing the interpretation of $\kappa/2\pi$ as 'thermodynamic' temperature. In the dimensionless variable $\varepsilon_\kappa = E/\kappa$ the vacuum spectrum reads

$$S_{\text{hyp}}(\varepsilon_\kappa) = \frac{\varepsilon_\kappa^3}{2\pi^2(e^{2\pi\varepsilon_\kappa} - 1)} . \tag{36}$$

3. $|\kappa| < |\tau_1|$, $\tau_2 = 0$, $\rho^2 = \tau_1^2 - \kappa^2$ (helical worldlines)

The excitation spectrum is an analytic function corresponding to the case 4 below only in the limit $\kappa \gg \rho$

$$S_{\text{hel}}(\varepsilon_\rho) \xrightarrow{\kappa/\rho \to \infty} S_{3/2-\text{parab}}(\varepsilon_\kappa) . \tag{37}$$

Letaw plotted the numerical integral $S_{\text{hel}}(\varepsilon_\rho)$, where $\varepsilon_\rho = E/\rho$ for various values of κ/ρ.

4. $\kappa = \tau_1$, $\tau_2 = 0$,   (if spatially projected: semicubical parabolas $y = \frac{\sqrt{2}}{3}\kappa x^{3/2}$)

The excitation spectrum is analytic, and since there are two equal curvature invariants one can use the dimensionless energy variable ε_κ

$$S_{3/2-\text{parab}}(\varepsilon_\kappa) = \frac{\varepsilon_\kappa^2}{8\pi^2\sqrt{3}}e^{-2\sqrt{3}\varepsilon_\kappa} . \tag{38}$$

It is worth noting that $S_{3/2-\text{parab}}$, being a monomial times an exponential, is quite close to the Wien-type spectrum $S_W \propto \varepsilon^3 e^{-\text{const}.\varepsilon}$.

5. $|\kappa| > |\tau_1|$, $\tau_2 = 0$, $\sigma^2 = \kappa^2 - \tau_1^2$ (if spatially projected: catenaries $x = \kappa \cosh(y/\tau)$)

In general, the catenary spectrum cannot be found analytically. It is an intermediate case, which for $\tau/\sigma \to 0$ tends to S_{hyp}, whereas for $\tau/\sigma \to \infty$ tends toward $S_{3/2-\text{parab}}$

$$S_{\text{hyp}}(\varepsilon_\kappa) \xleftarrow{0 \leftarrow \tau/\sigma} S_{\text{catenary}}(\varepsilon_\sigma) \xrightarrow{\tau/\sigma \to \infty} S_{3/2-\text{parab}}(\varepsilon_\kappa) . \tag{39}$$

6. $\tau_2 \neq 0$ (rotating worldlines uniformly accelerated normal to their plane of rotation)

The excitation spectrum is given in this case by a two-parameter set of curves. These trajectories are a superposition of the constant linearly accelerated motion and uniform circular motion. The corresponding vacuum spectra have not been calculated by Letaw, not even numerically.

Thus, only the hyperbolic worldlines, having just one nonzero curvature invariant, allow for a Planckian excitation spetrum and lead to a strictly one-to-one mapping between the curvature invariant κ and the 'thermodynamic' temperature ($T_U = \kappa/2\pi$). The excitation spectrum due to semicubical parabolas can be fitted by Wien-type spectra, the radiometric parameter corresponding to both curvature and torsion. The other stationary cases, being nonanalytical, lead to approximate determination of the curvature invariants, defining locally the classical worldline on which a relativistic quantum particle moves.

One can easily infer from these conclusions the reason why the Unruh effect became so prominent with regard to the other five types of stationary Frenet-Serret scalar spectra.

I mention that Letaw introduced the terminology *ultratorsional*, *paratorsional*, *infratorsional*, and *hypertorsional* for the scalar stationary cases **3**-**6**.

1.2. **The Electromagnetic Vacuum Noise**

For the case of homogeneous electromagnetic field, Honig *et al* proved that the FS scalars remain constant along the worldline of a charged particle, whereas the FS vectors obey a Lorentz force equation of the form

$$\dot{u}^{\mu} = \bar{F}^{\mu}_{\nu} u^{\nu} \tag{40}$$

with $F_{\mu\nu} = \lambda \bar{F}^{\mu}_{\nu}$, the electromagnetic field tensor, $\lambda = e/mc^2$, and $\dot{u}^{\mu}$ the four-velocity of the particle.

A remarkable physical interpretation of the FS invariants in terms of the Lorentz invariants of the electromagnetic field was established by Honig *et al*

$$\kappa^2 - \tau_1^2 - \tau_2^2 = \lambda^2(E^2 - H^2) , \qquad \kappa\tau_2 = -\lambda^2(\vec{E}\cdot\vec{H}) \tag{41}$$

These beautiful results passed quite unnoticed until now. They could be used for a geometric transcription of homogeneous electromagnetism and therefore for the geometric 'calibration' of electromagnetic phenomena.

The FS formalism has not been used for the electromagnetic vacuum noise. Other approaches have been undertaken for this important case of which a rather complete one is due to Hacyan and Sarmiento that is briefly presented in the following.

The Hacyan-Sarmiento approach

Starting with the expression for the electromagnetic energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{16\pi} \left(4F_{\mu\alpha} F^{\alpha}_{\nu} + \eta_{\mu\nu} F_{\lambda\beta} F^{\lambda\beta} \right) , \tag{42}$$

Hacyan–Sarmiento define the electromagnetic two-point Wightman functions as follows

$$D^+_{\mu\nu}(x,x') \equiv \frac{1}{4} \left(4F^{\alpha}_{(\mu}(x) F_{\nu)\alpha}(x') + \eta_{\mu\nu} F_{\lambda\beta}(x) F^{\lambda\beta}(x') \right); \tag{43}$$

$$D^-_{\mu\nu}(x,x') \equiv D^+_{\mu\nu}(x',x). \tag{44}$$

This may be viewed as a variant of the "point-splitting" approach advocated by DeWitt. Moreover, because of the properties

$$\eta^{\mu\nu} D^{\pm}_{\mu\nu} = 0, \qquad D^{\pm}_{\mu\nu} = D^{\pm}_{\nu\mu}, \qquad \partial_\nu D^{\pm\nu}_\mu = 0, \tag{45}$$

the electromagnetic Wightman functions can be expressed in terms of the scalar Wightman functions as follows

$$D^{\pm}_{\mu\nu}(x,x') = c \, \partial_\mu \, \partial_\nu \, D^{\pm}(x,x'), \tag{46}$$

where c is in general a real constant depending on the case under study. This shows that from the standpoint of their vacuum fluctuations the scalar and the electromagnetic fields are not so different.

Now introduce sum and difference variables

$$s = \frac{t+t'}{2}; \qquad \sigma = \frac{t-t'}{2}. \tag{47}$$

Using the Fourier transforms of the Wightman functions

$$\tilde{D}^{\pm}(\omega,s) = \int_{-\infty}^{\infty} d\sigma \, e^{-i\omega\sigma} \, D^{\pm}(s,\sigma), \tag{48}$$

where ω is the frequency of zero-point fields, the *particle number density of the vacuum seen by the moving detector* and the *spectral vacuum energy density per mode* are given by

$$n(\omega,s) = \frac{1}{(2\pi)^2\omega} \left[\tilde{D}^+(\omega,s) - \tilde{D}^-(\omega,s) \right], \tag{49}$$

$$\frac{de}{d\omega} = \frac{\omega^2}{\pi} \left[\tilde{D}^+(\omega,s) + \tilde{D}^-(\omega,s) \right]. \tag{50}$$

Circular worldline

The most important application of these results is to a uniformly rotating detector whose proper time is s and angular speed is ω_0 in motion along the circular world line

$$x^\alpha(s) = (\gamma s, R_0 \cos(\omega_0 s), R_0 \sin(\omega_0 s), 0), \tag{51}$$

where R_0 is the rotation radius in the inertial frame, $\gamma = (1-v^2)^{-1/2}$, and $v = \omega_0 R_0/\gamma$. In this case there are two Killing vectors $k^\alpha = (1,0,0,0)$ and $m^\alpha(s) = (0, -R_0 \sin(\omega_0 s), R_0 \cos(\omega_0 s), 0)$. Expressing the Wightman functions in terms of these two Killing vectors, HS calculated the following physically observable spectral quantities (i.e., those obtained after subtracting the inertial zero-point field contributions):

- *The spectral energy density*

$$\frac{de}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} \frac{v^3 w^2}{w^2 + (2\gamma v)^2} h_\gamma(w), \tag{52}$$

- *The spectral flux density*

$$\frac{dp}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} 4v^4 \, k_\gamma(w), \tag{53}$$

- *The spectral stress density*

$$\frac{ds}{d\omega} = \frac{\gamma^3}{2\pi^3 R_0^3} \frac{\omega^2 + (\gamma v \omega_0)^2}{\omega^2} \frac{v^3 w^2}{w^2 + (2\gamma v)^2} j_\gamma(w). \tag{54}$$

The ratio $(\omega^2 + (\gamma v \omega_0)^2)/\omega^2$ is a density-of-states factor introduced for convenience.

The Hacyan-Sarmiento variables are

$$w = \frac{2\omega}{\omega_0} \qquad x = \frac{\sigma \omega_0}{2}$$

$h_\gamma(w)$, $k_\gamma(w)$, and $j_\gamma(w)$ are the following cosine-Fourier transforms

$$h_\gamma(w) \equiv \int_0^\infty \left(\frac{N_h(x,v)}{\gamma^2 [x^2 - v^2 \sin^2 x]^3} - \frac{3}{x^4} + \frac{2\gamma^2 v^2}{x^2} \right) \cos(wx)\, dx; \tag{55}$$

$$k_\gamma(w) \equiv - \int_0^\infty \left(\frac{N_k(x,v)}{\gamma^2 [x^2 - v^2 \sin^2 x]^3} - \frac{3}{x^4} - \frac{\gamma^2}{6x^2} \right) \cos(wx)\, dx; \tag{56}$$

$$j_\gamma(w) \equiv \int_0^\infty \left(\frac{1}{\gamma^4 [x^2 - v^2 \sin^2 x]^2} - \frac{1}{x^4} + \frac{2\gamma^2 v^2}{3x^2} \right) \cos(wx)\, dx. \tag{57}$$

The numerators $N_h(x,v)$ and $N_k(x,v)$ are given by

$$N_h(x,v) = (3 + v^2)x^2 + (v^2 + 3v^4)\sin^2 x - 8v^2 x \sin x; \tag{58}$$

$$N_k(x,v) = x^2 + v^2 \sin^2 x - (1 + v^2)x \sin x. \tag{59}$$

Ultra-relativistic limit: $\gamma \gg 1$

The functions $H_\gamma = \frac{v^3 w^2}{w^2 + (2\gamma v)^2}\, h_\gamma(w)$, $\quad K_\gamma = 4v^4\, k_\gamma(w)$, $\quad J_\gamma = \frac{v^3 w^2}{w^2 + (2\gamma v)^2}\, j_\gamma(w)$, all have the following scaling property $X_{k\gamma}(kw) = k^3 X_\gamma(w)$, where k is an arbitrary constant, and $X = H, K, J$, respectively. This is the same scaling property as that of a Planckian distribution with a temperature proportional to γ.

Nonrelativistic limit: $\gamma \ll 1$

A detailed discussion of the nonrelativistic limit for the scalar case has been provided by Kim, Soh, and Yee [8], who used the parameters v and ω_0, and not acceleration and speed as used by Letaw and Pfautsch for the circular scalar case [9]. They obtained a series expansion in velocity

$$\frac{de}{d\omega} = \frac{\omega^3}{\pi^2} \left[\frac{\omega_0}{\gamma \omega} \sum_{n=0}^\infty \frac{v^{2n}}{2n+1} \sum_{k=0}^n (-1)^k \frac{(n-k-\frac{\omega}{\gamma \omega_0})^{2n+1}}{k!\,(2n-k)!} \Theta\left(n - k - \frac{\omega}{\gamma \omega_0}\right) \right], \tag{60}$$

where Θ is the usual Heavyside step function. Thus, to a specified power of the velocity many vacuum harmonics could contribute; making the energy density spectrum quasi-continuous. For low frequencies the difference between the scalar and electromagnetic case is small. Besides, one can consider only the first few terms in the series as an already good approximation.

1.3. Non-stationary vacuum field noise

Non-stationary vacuum field radiation has a time-dependent spectral content requiring joint time and frequency information, i.e., one needs generalizations of the power spectrum concept. One can think of (i) tomographical processing and/or (ii) wavelet transforms. For instance, the recently proposed non-commutative tomography (NCT) transform $M(s; \mu, \nu)$ [12], if relativistically generalized, could be an attractive way of processing non-stationary signals. In the definition of M, s is just an arbitrary curve in the non-commutative time-frequency plane, while μ and ν are parameters characterizing the curve. The most simple examples are the axes $s = \mu t + \nu \omega$, where μ and ν are linear combination parameters. The non-commutative tomography transform is related to the Wigner–Ville quasi-distribution $W(t, \omega)$ by an invertible transformation and has the following useful properties

$$M(t; 1, 0) = |f(t)|^2, \tag{61}$$
$$M(\omega; 0, 1) = |f(\omega)|^2, \tag{62}$$

where f is the analytic signal which is simulated by M. Furthermore, employing M leads to an enhanced detection of the presence of signals in noise which has a small signal-to-noise ratio. The latter property may be very useful in detecting vacuum field radiation noises, which are very small 'signals' with respect to more common noise sources.

On the other hand, since in the quantum detector method, the vacuum autocorrelation functions are the essential physical quantities. In addition, according to various fluctuation-dissipation theorems these functions are related to the linear (equilibrium) response functions to an initial condition/vacuum. The fluctuation-dissipation approach has been developed and promoted by Sciama and collaborators [13]. In principle, the generalization of the fluctuation-dissipation theorem for some classes of out of equilibrium relaxational systems, such as glasses, looks also promising for the case of non-stationary vacuum noise. One can use a so-called two-time fluctuation-dissipation ratio $X(t, t')$ and write a modified fluctuation-dissipation relationship [14]

$$T_{\text{eff}}(t, t') \, R(t, t') = X(t, t') \, \frac{\partial \tilde{C}(t, t')}{\partial t'}, \tag{63}$$

where R is the response function and $\tilde{C}$ the autocorrelation function. The fluctuation-dissipation ratio is employed to perform the separation of scales. Moreover, T_{eff} are timescale-dependent quantities, making them promising for relativistic vacuum radiation, which corresponds naturally to out of equilibrium conditions.

The necessity of a generalized nonequilibrium formalism, beyond Unruh effect, has been emphasized from a different standpoint by Hu and Johnson [15].

2. DETECTION PROPOSALS

A number of model experiments to detect 'Unruh radiation' have been suggested in the last twenty years. The author has undertaken the task of reviewing most of them [10, 11]. Here we present un update of our previous reviews. Because the curvature thermodynamic temperature is given by $T_\kappa = \frac{\hbar}{2\pi ck}a$ this leads to $T_\kappa = 4 \cdot 10^{-23}\,a$ and one needs accelerations greater than $10^{20}g_\oplus$ to have 'thermal' effects of only a few Kelvins. On the other hand, one should focus below the Schwinger acceleration for copious spontaneous pair creation out of QED vacuum

$$a_{\mathrm{Schw}} \approx m_e c^3/\hbar \approx 10^{29}\,m/s^2 \approx 10^{28}\,g_\oplus\,, \tag{64}$$

There are indeed several physical settings in which accelerations can be achieved only a few orders below the Schwinger acceleration and forthcoming technological advances could test routinely those acceleration scales. The Unruh effect, if it exists, can be revealed as a tiny signal in the background of by far more powerful effects.

2.1. Unruh Effect in Storage Rings ($a \sim 10^{22}g_\oplus$, $T_\kappa = 1200\,K$)

J.S. Bell and J.M. Leinaas imagined the first laboratory phenomenon connected to the Unruh effect. During 1983-1987 they published a number of papers on the idea that the depolarising effects in electron storage rings could be interpreted in terms of Unruh effect [16, 17, 18, 19]. However, the incomplete radiative polarization of the electrons in storage rings has been first predicted in early sixties in the framework of QED by Sokolov and Ternov. Besides, the circular vacuum noise is not sufficiently "universal" since it always depends on both acceleration and velocity. This appears as a 'drawback' of the 'storage ring electron thermometry', not to mention the very intricate spin physics. Keeping these facts in mind, we go on with further comments, following the nontechnical discussion of Leinaas [20].

The circular acceleration in the LEP machine is $a_{LEP} = 3 \times 10^{22}g_\oplus$ corresponding to the Unruh temperature $T_U = 1200\,^oK$. It is a simple matter to show that an ensemble of electrons in a uniform magnetic field at a nonzero temperature will have a polarization expessed through the following hyperbolic tangent $P_U = \tanh(\frac{\pi g}{2\beta})$. For the classical value of the gyromagnetic factor ($g = 2$) and for highly relativistic electrons ($\beta = 1$), $P_U = \tanh\pi = 0.996$, beyond the limiting polarization of Sokolov and Ternov $P_{ST,\max} = \frac{8\sqrt{3}}{15} = 0.924$ [21].

On the other hand, the function

$$P_U(g) = \tanh(\frac{\pi g}{2\beta}) = \frac{1 - e^{-\pi g}}{1 + e^{-\pi g}} \tag{65}$$

is very similar, when plotted, to the function $P_{DK}(g)$, which is a combination of exponential and polynomial terms in the anomalous part of the gyromagnetic factor of the electron, and it was obtained through QED calculations by Derbenev and Kondratenko [22]

$$P_{eq}^{DK} = \frac{8}{5\sqrt{3}}\frac{<|\,\rho\,|^{-3}\,\hat{b}(\hat{n} - F_{DK}) >}{<|\,\rho\,|^{-3}\,(1 - 2/9(\hat{n}\cdot\hat{v})^2 + 11/18\,|\,F_{DK}\,|^2) >}\,, \tag{66}$$

where $F_{DK} = \gamma\frac{\partial\hat{n}}{\partial\gamma}$ is the spin-orbit coupling function, which takes into account the depolarizing effects of jumps between various trajectories differing from the reference closed orbit, ρ is the bending radius, $\hat{b}$ a unit vector along the transverse magnetic field component. The brackets indicate an average over the ring circumference and over the ensemble of particles in the beam. The unit vector $\hat{n}$ is the time-independent spin solution of the BMT equation, attached to each particle trajectory.

The difference between P_U and P_{DK} is merely a shift of the latter along the positive g-axis with about 1.2 units. As shown by Bell and Leinaas, when the Thomas precession of the electron is included in the spin Hamiltonian, a shift of 2 units is obtained for $P_U(g)$. This suggests a more careful treatment of spin effects arising when one is going from the lab system to the circulating coordinate frame. A new spin Hamiltonian was introduced by Bell and Leinaas with a more complicated structure of the field vector in the scalar product with the Pauli matrices. This complicated structure takes into account the classical external fields, the quantum radiation field and the fluctuations around the classical path. Within this more complete treatment, Bell and Leinaas were able to get, to linear order in the quantum fluctuations, a Thomas-like term and a third resonant term. The latter is directly related to the vertical fluctuations in the electron orbit, which are responsible for the spin transitions

$$P_{BL} = \frac{8}{5\sqrt{3}} \frac{1 - \frac{f}{6}}{1 - \frac{f}{18} + \frac{13}{360} f^2} \tag{67}$$

where the resonance factor f reads

$$f = \frac{2 a_g Q^2}{Q_z^2 - a_g^2 \gamma^2} . \tag{68}$$

Q_z is the vertical betatron tune, γ is the Lorentz factor and $a_g = (g-2)/2$ is the electron g anomaly. This induces an interesting variation of the beam polarization close to the resonance. As γ passes through it from below, the polarization first falls from 92% to -17%, and then it increases again to 99% before stabilizing to 92% . This is the only clear difference from the standard QED. Such resonances induced by the vertical fluctuations of the orbit have been considered before in the literature within classical spin diffusion models and focused strictly on their depolarizing effect. Their nature is related to the fact that the Fourier spectrum of the energy jumps associated with the quantum emission processes contains harmonics giving the usual resonance condition. As emphasized by Bell and Leinaas, a more direct experimental demonstration of the circular Unruh noise would be the measurement of the vertical fluctuations. However, this will clearly be a very difficult task since such fluctuations are among the smallest orbit perturbations. At the same time, the measurement of the polarization variation close to the narrow resonance, in particular the detection of transient polarizations exceeding the Sokolov-Ternov limiting one, will make us more confident in the claims of Bell and Leinaas. It is worth mentioning that the rapid passage through the resonance does not change the polarization, while a slow passage reverses it but does not change the degree of polarization. Therefore only an intermediate rate with respect to the quantum emission time scale of passing through the resonance will be appropriate for checking the transient BL effects. Barber and Mane [23] have shown that the DK and BL formalisms for the equilibrium degree of radiative electron polarization are not so different as they might look. They also obtained an even more general formula for P_{eq} than the DK and BL ones and from their formula they estimated as negligible the BL increase near the resonance. It reduces to the following substitution of the resonance factor

$$f \rightarrow f + \frac{2}{\gamma} . \tag{69}$$

The basic experimental data on spin depolarizing effects remain as yet those measured at SPEAR at energies around 3.6 GeV in 1983. Away from the resonant $\gamma's$ the maximum polarization of Sokolov and Ternov was confirmed [24] but no dedicated study of the polarization rates across resonances have been performed.

A paper of Cai, Lloyd and Papini [25] claims that the Mashhoon effect due to the spin-rotation coupling is stronger than the circular Unruh effect (spin-acceleration coupling) at all

accelerator energies in the case of a perfect circular storage ring. However, the comparison is not at all a straightforward one.

Bautista [26] solved the Dirac equation in Rindler coordinates with a constant magnetic field in the direction of acceleration and showed that the Bogolubov coefficients of this problem do not mix up the spin components. Thus there is no spin polarization due to the acceleration in this case.

In our opinion, the real importance of considering Unruh effect at storage rings is related to clarifying radiometric features of the synchrotron radiation [27]. There is a strong need to establish radiometric standards in spectral ranges much beyond those of the cavity/blackbody standards, and synchrotron radiation has already been considered experimentally from this point of view [28]. Quantum field thermality is intrinsically connected to the KMS condition. This is a well-known skew periodicity in imaginary time of Green's functions expressing the detailed balance criterion in field theory. However, the task is to work out in more definite terms the radiometric message of the KMS quantum/stochastic processes [29].

Recently, the spin-flip synchrotron radiation has been experimentally shown to be important in the hard part of the spectrum in the axial channeling of electrons in the energy range 35-243 GeV incident on a W single crystal [30]. This may revive the interest in the BL interpretation, especially in the cleaner planar channeling case [31].

2.2. **Unruh Effect and Geonium Physics** ($a \sim 10^{21} g_\oplus$, $T_\kappa = 2.4\,K$)

The very successful Geonium physics could help detecting the circular thermal-like vacuum noise. The proposal belongs to J. Rogers [32] being one of the most attractive. The idea of Rogers is to place a small superconducting Penning trap in a microwave cavity. A single electron is constrained to move in a cyclotron orbit around the trap axis by a uniform magnetic field (Rogers figure is B = 150 kGs). The circular proper acceleration is $a = 6 \times 10^{21} g_\oplus$ corresponding to T = 2.4 K. The velocity of the electron is maintained fixed ($\beta = 0.6$) by means of a circularly polarized wave at the electron cyclotron frequency, compensating also for the irradiated power. The static quadrupole electric field of the trap creates a quadratic potential well along the trap axis in which the electron oscillates. The axial frequency is 10.5 GHz (more than 150 times the typical experimental situation [33]) for the device scale chosen by Rogers. This is the measured frequency since it is known [33] that the best way of observing the electron motion from the outside world (Feynman's "rest of the Universe") is through the measurement of the current due to the induced charge on the cap electrodes of the trap, as a consequence of the axial motion of the electron along the symmetry axis. At 10.5 GHz the difference in energy densities between the circular noise and the universal linear noise are negligible (see Fig. 2 in Rogers' work). Actually, Rogers used the parametrization for the spectral energy density of a massless scalar field as given by Kim, Soh and Yee [8] that he wrote in the form

$$\frac{de}{d\omega} = \frac{\hbar}{\pi^2 c^3}\left[\frac{\omega^3}{2} + \gamma\omega_c^3 x^2 \sum_{n=0}^{\infty}\frac{\beta^{2n}}{2n+1} + \sum_{k=0}^{n}(-1)^k\frac{n-k-x}{k!(2n-k)}\Theta[n-k-x]\right] \quad (70)$$

where γ is the relativistic gamma factor, $x = \omega/\gamma\omega_c$, and $\omega_c = eB/\gamma mc$ is the cyclotron frequency. The power spectral density at the axial frequency is only $\partial P/\partial f = 0.47 \cdot 10^{-22}$ W/Hz, and may be assumed to be almost the same as the electromagnetic spectral energy density. This power is resonantly transfered to the TM_{010} mode of the microwave cavity and a most sensible cryogenic GaAs field-effect transistor amplifier should be used to have an acceptable signal-to-noise ratio of S/N = 0.3. According to Rogers, the signal can be distinguished from the amplifier noise in about 12 ms.

In conclusion, very stringent conditions are required in the model experiment of Rogers. Top electronics and cryogenic techniques are involved as well as the most sophisticated geonium methods. Taking into account the high degree of precision attained by geonium techniques, one may think of Rogers' proposal as one of the most feasible. The critique of this proposal is similar to that in storage rings [34] given that the circular Unruh effect is not universal, depending also on the electron velocity. In addition, Levin, Peleg, and Peres [35] studied the Unruh effect for a massless scalar field enclosed in a two-dimensional circular cavity concluding that the effects of finite cavity size on the frequencies of normal modes of the cavity (Casimir effect) ignored by Bell *et al* and by Rogers are in fact quite important.

Cylindrical Penning Traps

A better experimental setting for detecting vacuum noises by means of a trapped quantum detector (electron) may well be the cylindrical Penning trap, for which the trap itself is a microwave cavity [36]. In this case small slits incorporating choke flanges divide high-conductivity copper cavity walls into the required electrode Penning configuration, including two compensation electrodes. The driven axial resonance for this configuration has already been observed with almost the same signal-to-noise ratio as in hyperbolic Penning traps. By means of these cylindrical cavity traps, a more direct coupling to the cavity modes may be achieved, especially in the weak coupling regime. Here the cyclotron oscillator and the cavity mode cannot form normal modes, and therefore other nonlinear effects are not coming into play. The cylindrical TM_{010} mode is essentially a zero-order Bessel function in the radial direction and has no modes along the z axis. The price to pay in the case of the cylindrical trap is a loss of control on the quality of the electrostatic quadrupole potential.

2.3. Unruh Effect and Nonadiabatic Casimir Effect ($a \sim 10^{20} g_{\oplus}, T \sim 1 K$)

An experimental equivalent of a *fast moving mirror* might be a *plasma front* created when a gas is suddenly photoionized. This is the proposal of Yablonovitch.[37] The argument is that the phase shift of the zero-point electromagnetic field transmitted through a plasma window whose index of refraction is falling with time (from 1 to 0) is the same as when reflected from an accelerating mirror. Consider the case of hyperbolic motion. Since the velocity is

$$v = c \tanh(a\tau/c) \tag{71}$$

where τ is the observer's proper time, the Doppler shift frequency will be

$$\omega_D = \omega_0 \sqrt{\frac{1 - v/c}{1 + v/c}} = \omega_0 \exp(-a\tau/c) \tag{72}$$

and consequently a plane wave of frequency ω_0 turns into a wave with a time-dependent frequency. Such waves are called chirped waves in nonlinear optics and acoustics. Eq. (72) represents an *exponential chirping* valid also for black holes. For an elementary discussion of Doppler shift for accelerated motion see Ref.[38]. It is worthwhile to mention that in the semiclassical treatment of black hole physics one is usually dealing with chirped signals, since the WKB functions are generally of variable wavelength, and by meeting supplementary conditions on their derivatives they are made to look as much as possible like fixed linear combinations of plane waves. On the other hand, in the case of wave packets one is always working with the average frequency of the wave packets (see the second paper of Jacobson [?] or the paper of Frolov and Novikov on the dynamical origin of black hole entropy [39]).

The technique of producing plasma fronts/windows in a gas by laser breakdown, and the associated frequency upshifting phenomena (there are also downshifts) of the electromagnetic

waves interacting with such windows, are well settled since about twenty years. Blue shifts of about 10% have been usually observed in the transmitted laser photon energy.

In his paper, Yablonovitch works out a very simple model of a *linear* chirping due to a refractive index linearly decreasing with time, $n(t) = n_0 - \dot{n}t$, implying a Doppler shift of the form $\omega \rightarrow \omega[1 + \frac{\dot{n}}{n}t] \sim \omega[1 + \frac{a}{c}t]$. To have accelerations $a = 10^{20}g_\oplus$ the laser pulses should be less than 1 picosecond. Even more promising may be the nonadiabatic photoionization of a semiconductor crystal in which case the refractive index can be reduced from 3.5 to 0 on the timescale of the optical pulse. As discussed by Yablonovitch, the pump laser has to be tuned just below the Urbach tail of a direct-gap semiconductor in order to create weakly bound virtual electron-hole pairs. These pairs contribute a large reactive component to the photocurrent since they are readily polarized. The background is due to the bremsstrahlung emission produced by real electron-hole pairs, and to diminish it one needs a crystal with a big Urbach slope (the Urbach tail is an exponential behavior of the absorption coefficient).

Ambiguity of the interpretation
Yablonovitch remarked that the experimental interpretation is highly ambigous. There are two types of reasoning:

(i) technologically-oriented interpretations

1. A single-cycle microwave squeezing

2. An inverse quadratic electro-optic effect with zero-point photons as input waves

(ii) concept-oriented interpretations

3. Nonadiabatic Casimir effect

4. Unruh effect

Finally, one should notice the difference between the laboratory and the black hole/hyperbolic chirping. The former is *linear*, whereas the latter is *exponential*.

The 'plasma window' of Yablonovitch was criticized in the important paper by Dodonov, Klimov, and Nikonov [DKN] [40] on the grounds that we are not in the case of exponentially small reflection coefficient as required to get a Planck spectrum from vacuum fluctuations. At the general level, one may argue that nonstationary Casimir effects may produce some deformed Planck distributions, and only in particular cases purely Planck distributions. As a matter of fact, depending on the nonstationarity, one may obtain very peculiar photon spectra, and this might be of great interest in applied physics. DKN showed explicitly that an exponential 'plasma window', for which presumably the modulation 'depth' is the effective Unruh temperature, does not produce a Planck spectrum. However, for a parametric function displaying the symmetric Epstein profile one can get in the adiabatic limit a 'Wien's spectrum' with the effective temperature given by the logarithmic derivative of the variable magnetic permeability with respect to time. According to DKN this corresponds to a 'dielectric window' and not to a 'plasma window'. The experimental realization of nonstationary Casimir effects are either resonators with moving walls, as first discussed by Moore,[41] or resonators with time-dependent refractive media as discussed by DKN. On the lines of Yablonovitch, Hizhnyakov [42] studied the sudden changes of the refractive index caused by the excitations of a semiconductor near a band-to-band transition in the infrared by a synchroneously pumped subpicosecond dye laser, and also refered to the anology

with Hawking and Unruh effects. In 1994, C.K. Law [43] combined the moving walls of Moore with the dielectric medium with time-varying permittivity in a one-dimensional electromagnetic resonant cavity. In this way, he obtained an effective quadratic Hamiltonian, which is always required when we want to discuss nonstationary 'particle production' effects.

2.4. Unruh effect and sonoluminescence

A few years ago, Eberlein elaborated more on Schwinger's interpretation of sonolumines-cence in terms of zero point fluctuations and darely mentioned Unruh effect [44]. She claimed that sonoluminescence could be interpreted as a generalized Unruh's effect that goes beyond the perfect mirrors restriction. In particular, whenever *an interface between two dielectrics or a dielectric and the vacuum moves noninertially photons are created*.

Choosing as model profile for the time-dependence of the bubble radius about the collapse

$$R^2(t) = R_0^2 - (R_0^2 - R_{\min}^2)\frac{1}{(t/\gamma)^2 + 1} , \tag{73}$$

Eberlein obtained for bubble radius R much greater than the wavelengths of the light emitted the thermal-like spectral density

$$S(\omega) = \frac{(n^2 - 1)^2}{64n^2} \frac{\hbar}{c^4\gamma}(R_0^2 - R_{\min}^2)^2 \omega^3 e^{-2\gamma\omega} , \tag{74}$$

using only zero-temperature quantum field calculations.calculations. She also calculated the total energy radiated during one acoustic cycle. She obtained $\sim 2 \cdot 10^{-13}$ in a lapse of 1 femtosecond, claiming that this roughly corresponds to the observed number of photons. Her results were strongly contended by many comments. In particular, Milton [45] used the Larmor formula $P = \frac{2}{3}\frac{(\ddot{d})^2}{c^3}$ with the electric dipole moment $d \sim eR$ to estimate the classical Larmor energy radiated in the flash of bubble collapse $E \sim \alpha\hbar c\frac{a^2}{c^3\tau^3}$. This leads to only 10 eV for a flash of 1 femtosecond. To get 10^7 eV, the normal energy of the flash, one needs a time scale as short as 10^{-17} s, if Larmor radiation alone is invoked. Milton noted that models are required in which the velocity of the radius is nonrelativistic while the acceleration becomes very large, and that this situation occurs during the formation of shocks

2.5. Unruh Effect and Squeezing in Fibers

Grishchuk, Haus, and Bergman [46] discussed in a Phys. Rev. D paper a nonlinear Mach-Zhender configuration to generate radiation through the optical squeezing of zero-point fluctuations interacting with a moving index grating. The squeezed vacuum optical radiation has been generated experimentally by separating the pump light from the squeezed fluctuations in the MZ interferometric geometry. The similarity with Unruh effect is mentioned in Section VIII of that paper. There are two conditions to accomplish a strong optical squeezing in the laboratory: one needs to supress classical noise and phase match the vacuum wave with the exciting source. These two conditions are very well satisfied by working with fibers.

2.6. **Unruh Effect and Channeling** ($a \sim 10^{30} g_\oplus$, $T_\kappa \sim 10^{11} K$?)

Relativistic particles can acquire extremely high transverse accelerations when they are channeled through crystals. Darbinian and collaborators [47] related this physical setting to Unruh radiation.

First YERPHY Proposal
The idea is to measure the *Unruh radiation emitted in the Compton scattering of the channeled particles with the Planck spectrum of the inertial crystal vacuum.* The main argument is that the crystallographic fields are acting with large transverse accelerations on the channeled particles. The estimated transverse proper acceleration for positrons channeled in the (110) plane of a diamond crystal is $a = 10^{25}\gamma$ cm/s^2, and at a $\gamma = 10^8$ one could reach 10^{33} cm/s$^2 = 10^{30}g_\oplus$. Working first in the particle instantaneous rest frame, the Erevan group derived the spectral angular distribution of the Unruh photons in that frame. By Lorentz transformation to the lab system they got the number of Unruh photons per unit length of crystal and averaged over the channeling diameter. At about $\gamma = 10^8$ the Unruh intensity, i.e., the intensity per unit pathlength of the Compton scattering on the Planck vacuum spectrum becomes comparable with the Bethe-Heitler bremsstrahlung ($dN_\gamma/dE \propto 1/E$, and mean polar emission angle $\theta = 1/\gamma$).

Incidentally, there is a parallel with some experiments [48, 49, 50] performed at LEP, where the scattering of the LEP beam from the thermal photon background in the beam pipe has been measured (the black body photons emitted by the walls of the pipe have a mean energy of 0.07 eV). It is considered as a fortunate case that the effect is too small to affect the lifetime of the stored beams.

Second YERPHY Proposal
In another work of the armenian group [51] the same type of calculations was applied to get an estimate of the *Unruh radiation generated by TeV electrons in a uniform magnetic field as well as in a laser field.* The Unruh radiation becomes predominant over the synchrotron radiation only when $\gamma = 10^9$ for $H = 5 \cdot 10^7 Gs$ and consequently it is practically impossible to detect it at present linear colliders. Supercolliders with bunch structure capable of producing magnetic fields of the order $10^9 G$ are required. Pulsar magnetospheres are good candidates for considering such a Unruh radiation.

A circularly polarized laser field seems more promising since in this case the Unruh radiation could be detectable at lower magnetic fields and energies ($\gamma = 10^7$). This is due to the fact that the proper centripetal acceleration of the electron is $a = 2\omega\gamma\eta\sqrt{1+\eta^2}$, where ω is the frequency of the electromagnetic wave, and $\eta = e\varepsilon/m\omega$ (ε being the amplitude of the field).

2.7. **Unruh Radiation and Ultraintense Lasers** ($a \sim 10^{25} g_\oplus$, $T_\kappa = 1.2\, 10^6 K$)

The most recent proposal is to get an Unruh signal in electron Petawatt-class laser interaction. It has been put forth by Chen and Tajima in 1999 [52].
Uniform acceleration through the usual quantum vacuum (Minkowski vacuum) of the electromagnetic field distorts the two-point function of the zero-point fluctuations (ZPF) in such a way that

$$\langle E_i(\sigma_-)E_j(\sigma_+)\rangle = \frac{4\hbar}{\pi c^3}\,\delta_{ij}\,\frac{(a/c)^4}{\sinh^4(a\tau/2c)}.$$ (75)

The main point of Tajima and Chen is to introduce the so-called laser parameter (see below) in this formula. In their approach in the leading order the accelerated electron is assumed "classical", with well-defined acceleration, velocity, and position. This allows to introduce a Lorentz transformation so that the electron is described in its instantaneous proper frame. Also at this level the linearly accelerated electron will execute a classical Larmor radiation. As a response to the Larmor radiation, the electron reacts to the vacuum fluctuations with a nonrelativistic quivering motion in its proper frame that triggers additional radiation. The interaction Hamiltonian can be written as

$$H_{int} = -\frac{e}{mc}\vec{p}\cdot\vec{A} = -e\vec{x}\cdot\vec{E} . \tag{76}$$

The probability of the emission of a photon with energy $\omega = \varepsilon' - \varepsilon$ is

$$N(\omega) = \frac{1}{\hbar^2}\int d\sigma \int d\tau |\langle 1_{\vec{k}}, \varepsilon'|H_{\text{int}}|\varepsilon, 0\rangle|^2$$

$$= \frac{e^2}{\hbar^2}\sum_{i,j}^{3}\int d\sigma \int d\tau e^{-i\omega\tau}\langle x_i(\sigma)x_j(\sigma)\rangle\langle E_i(\sigma_-)E_j(\sigma_+)\rangle , \tag{77}$$

where $\sigma_- = \sigma - \tau/2$ and $\sigma_+ = \sigma + \tau/2$ are combinations of σ and τ, the absolute and relative proper time, respectively. The τ dependence of the position operator has been extracted to the phase due to a unitary transformation. The last bracket is the autocorrelation function for the fluctuations of the electric field in the vacuum.

The laser is treated as a plane EM wave and since the interest is in a *quasilinear acceleration*, two identical, counterpropagating plane waves are considered in order to provide a standing wave.

$$E_x = E_0[\cos(\omega_0 t - k_0 z) + \cos(\omega_0 t + k_0 z)] , \tag{78}$$
$$B_y = E_0[\cos(\omega_0 t - k_0 z) - \cos(\omega_0 t + k_0 z)] , \tag{79}$$

To get the quasilinear acceleration and hopes for the Unruh signal, one should carefully tune the 'laser' at the nodal points $k_0 z = 0, \pm 2\pi, ...$, where $B_y = 0$ identically for all times and E_x takes the maximum value. At $z = 0$, Chen and Tajima find

$$\gamma\beta_x = 2a_0\sin(\omega_0 t) \tag{80}$$

with

$$\gamma = \sqrt{1 + 4a_0^2\sin^2\omega_0 t} , \quad \text{and} \quad a_0 = \frac{eE_0}{mc\omega_0} ,$$

the relativistic γ factor and the dimensionless laser strength parameter, respectively.

The proper acceleration, which is related to that in the laboratory frame by $a = \gamma^3 a_{\text{lab}}$, is thus

$$a = 2ca_0\omega_0\cos\omega_0 t , \tag{81}$$

Autocorrelation function

One has to look for the transformation between laboratory and proper spacetimes. As $d\tau = dt/\gamma$, in the limit $a \gg 1$ one finds

$$\sin\omega_0 t = \tanh(2a_0\omega_0\tau)/\sqrt{1 + 4a_0^2\text{sech}^2(2a_0\omega_0\tau)}$$

$$\sin k_0 x = 2a_0\cos\omega_0 t/\sqrt{1 + 4a_0^2} . \tag{82}$$

For periodic motion, it is sufficient to examine the cycle $-\pi/2 \le \omega_0 t \le \pi/2 \rightarrow \pi/a_0 \le \omega_0 \tau \le \pi/a_0$. Within the limit where $4a_0^2 \text{sech}^2(2a_0\omega_0\tau) \gg 1$, the above equation reduces to

$$\sin\omega_0 t \approx \frac{1}{2a_0}\sinh(2a_0\omega_0\tau) \ ,$$

$$\cos k_0 x \approx \frac{1}{2a_0}\sinh(2a_0\omega_0\tau) \ . \tag{83}$$

In the limit of small arguments, these can be readily recognized as conformal transformations of the Rindler transformation for constant acceleration. Thus, the experiment corresponds to a quasi-constant acceleration and therefore one can use the known formula

$$\langle E_i(\sigma - \tau/2)E_j(\sigma + \tau/2)\rangle = \delta_{ij}\frac{4\hbar}{\pi c^3}(2a_0\omega)^4\text{csch}^4(a_0\omega_0\tau) \ . \tag{84}$$

From this, with some further mathematical tricks one arrives at the basic relationships

$$\frac{dN(\omega)}{d\sigma} = \frac{1}{2\pi}\frac{e^2}{\hbar c^3}(2a_0\omega_0)^3\langle x^2\rangle \int_{-\infty}^{+\infty} ds\, e^{-is\omega/a_0\omega_0}\text{csch}^4(s - i\varepsilon) \ , \tag{85}$$

$$\frac{dI_U}{d\sigma} = \int_{\omega_0}^{\infty}\hbar d\omega\frac{dN(\omega)}{d\sigma} \approx \frac{12}{\pi}\frac{r_e\hbar}{c}(a_0\omega_0)^3\log(a_0/\pi) \ . \tag{86}$$

The above result applies to an accelerated electron located exactly at $z = 0$. At the vicinity of this point, e.g., $k_0 z = \varepsilon \ll 1$, there is a nonvanishing magnetic field $|B_y| = \varepsilon|E_x|$, which induces a $\beta_z \approx O(\varepsilon)$ in addition to the dominant β_x. Nevertheless, the proper acceleration is affected only to the order ε^2

$$a \approx 2c\omega_0 a_0[1 - O(\varepsilon^2)]$$

For electrons farther away where the decrease of acceleration becomes more significant, both the Unruh and the background Larmor radiations will decrease much more rapidly due to their strong dependences on acceleration. Thus only the origin and its immediate surroundings are important within this proposal.

Unruh Radiation versus Larmor Radiation

At the classical level, the same linear acceleration induces a Larmor radiation.

In addition the photon $\vec{k}$ space that they are interested in detecting is along the direction of acceleration where the Larmor radiation is the weakest. Therefore the two radiations can be treated as independent processes without interference.

The total Larmor radiation power is

$$\frac{dI_L}{dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right) = \frac{8}{3}r_e mca_0^2\omega_0^2\cos^2(\omega_0 t) \ . \tag{87}$$

and the total energy radiated during each laser half-cycle is

$$\Delta I_L = \frac{4\pi}{3} r_e mc a_0^2 \omega_0 \ .$$

On the other hand, the Unruh radiation is significant over a reduced proper time period

$$\omega_0 \Delta\sigma \geq O(1/a_0)$$

Nevertheless, within this time the electron has become relativistic, with $\gamma \sim a_0$. As a result, the total energy radiated in the lab frame, i.e., $\Delta I_U \sim (dI_U)/(d\sigma)\gamma\Delta\sigma$, is $\Delta I_U \sim \frac{12}{\pi}(r_e\hbar/c)a_0^3\omega_0^2 \log(a_0/\pi)$. Thus the relative yield is

$$\frac{\Delta I_U}{\Delta I_L} \sim \frac{9}{\pi^2}\frac{\hbar\omega_0}{mc^2} a_0 \log(a_0/\pi) \ . \tag{88}$$

Since $a_0 \propto 1/\omega_0$, the relative yield is not sensitive to the laser frequency.

1. *Petawatt-class laser*: $\omega_0 \sim 2 \cdot 10^{15} \ \mathrm{sec}^{-1}$ and $a_0 \sim 100 \to \frac{\Delta I_U}{\Delta I_L} \sim 3 \cdot 10^{-4}$

2. *Free-electron laser-driven coherent X-ray source*: $\hbar\omega_0 \sim 10 \ \mathrm{keV}$ and $a_0 \sim 10 \to \frac{\Delta I_U}{\Delta I_L} \sim 1$

Blind spot of Larmor radiation

The Unruh 'thermal fluctuations' are considered isotropic in the electron's proper frame resulting in an isotropic induced radiation signal in the same frame. But since at each half cycle the electron rapidly becomes relativistic, with $\gamma \sim a_0$, the Unruh radiation is boosted along the direction of polarization (x axis) in the lab frame. Furthermore, as we have discussed above, the autocorrelation function, and therefore the Unruh signal, tend to diminish more rapidly than that from Larmor within the laser half cycle. This should induce a sharper time structure for the former.

Transforming the Unruh radiation power back to the lab frame with $\gamma \sim a_0$, the angular distribution in the small angle expansion becomes

$$\frac{d^2 I_U}{dt d\Omega} = \frac{4}{\pi^2}\frac{r_e\hbar}{c}\frac{\omega_0^3 a_0^3}{[1+a_0^2\theta^2]^3} \ . \tag{89}$$

The Larmor radiation is polarized and its angular distribution in the small (θ,ϕ) polar-angle expansion is

$$\frac{d^2 I_L}{dt d\Omega} = \frac{8 r_e mc a_0^2 \omega_0^2}{[1+a_0^2\theta^2]^3}\left[1 - \frac{4a_0^2\theta^2(1-\phi^2)}{[1+a_0^2\theta^2]^2}\right] \ . \tag{90}$$

The Larmor power is minimum at $(\theta,\phi) = (1/a_0\pi,0)$, where the angular distribution is zero. Consider a detector which covers an azimuthal angle $\Delta\phi = 10^{-3}$ around the "blind spot", and an opening polar angle, $\Delta\theta \ll 1/a_0$. Then the partial radiation power for the Unruh signal would dominate over that for the Larmor within this solid angle. A plot confirming this *blind spot* has been drawn by Rosu [53], whereas Visser [54] gave some interesting comments on the proposal of Chen and Tajima. I transcript here the following rethoric questions of Visser that he present as a sociology/linguistic issue: "If you ultimately succeed in seeing this ZPF-induced modification to Larmor radiation, should you really call it the Unruh effect? Or should you just call it basic quantum field theory? After all you are not directly measuring the Unruh temperature itself."

How to create ultralow electrons

1. *Low energy photoelectrons* near the surface of a solid material.

2. *Laser-trapped and cooled electrons*. Laser-electron interactions occur in vacuum and there should be minimal additional background effects.

3. *Low temperature plasma*. The main background effect is the bremsstrahlung. Even in this case the Unruh signal wins if the plasma density is low enough. The cross section of bremsstrahlung for an unscreened hydrogen nucleus per unit photon energy is

$$d\chi/d\hbar\omega \sim \frac{16}{3}\alpha r_e^2 \ln(EE^{'}/mc^2\omega) .$$

The bremsstrahlung yield depends quadratically, whereas the Unruh signal linearly on the plasma density. The figure suggested by Chen & Tajima is $n_p \leq 10^{18}$ cm^{-3}, below which the bremsstrahlung background can be minimized.

## 2.8. Unruh effect and damping in a linear focusing channel

K.T. McDonald [55] applied recently the Unruh temperature formula for a rapid calculation of the damping in a linear focusing channel [LFC] §. He used the same idea some 15 years ago to reproduce Sands' results on the limits of damping of the phase volume of beams in electron storage rings.

---

§ The LFC is a transport system that confines the motion of charged particles along straight central rays by means of a potential quadratic in the transverse spatial coordinates.

3. RELATED TOPICS

3.1. **Unruh Effect and Anomalous Doppler Effect (ADE)**

When studied with the detector method, the Unruh effect for a detector with internal degrees of freedom is very close to the anomalous Doppler effect (ADE). This is because in both cases the quantum detector is radiating 'photons' while passing onto the upper level and not on the lower one [60].
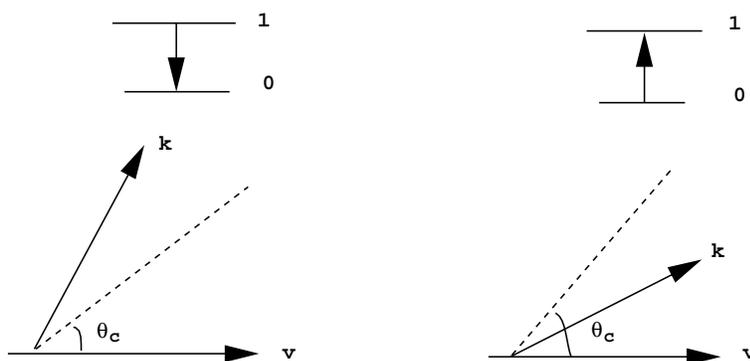
Figure 1. The normal and anomalous Doppler effects and the corresponding transitions.

It is worthwhile to note that the ADE-like concept was used by Unruh and Wald [56] without referring to it explicitly, when they considered the Unruh effect for a uniformly accelerated quantum detector looked upon from the inertial reference frame. Their main and well-known conclusion was that emission in an inertial frame corresponds to absorption from the Unruh's 'heat bath' in the accelerated frame. Essentially one may say the following.

(i) *For the observer placed in the noninertial frame the 'photon' is unobservable (it belongs to the left wedge in the Rindler case).*

(ii) *When the observer places himself in an inertial reference frame, he is able to observe both the excited quantum detector (furnishing at the same time energy to it) and the 'photons'. By writing down the energy-momentum conservation law he will be inclined to say that the 'photons' are emitted precisely when the detector is excited.*

> *Review of ADE Literature*

A quantum derivation of the formula for the Doppler effect in a medium has been given by Ginzburg and Frank already in 1947 [57] and more detailed discussion has been provided by Frank during 1970-1980 [58]. See also the 1993 review paper of Ginzburg [59].

Neglecting recoil, absorption, and dispersion (a completely ideal case) the elementary radiation events for a two-level detector with the change of the detector proper energy denoted by $\delta\varepsilon$ are classified according to the photon energy formula [60]

$$\hbar\omega = -\frac{\delta\varepsilon}{D\gamma} \tag{91}$$

where γ is the relativistic velocity factor ($\gamma > 1$) and D is the Doppler directivity factor

$$D = 1 - (vn/c)\cos\theta \tag{92}$$

The discussion of signs in Eq. (91) implies 3 cases as follows:

D$>$ 0 for normal Doppler effect (NDE, $\delta\varepsilon < 0$)
D $=$ 0 for Cherenkov effect (CE, $\delta\varepsilon = 0$, undetermined case)
D$<$ 0 for anomalous Doppler effect (ADE, $\delta\varepsilon > 0$).

Consequently, for a quantum system endowed with internal degrees of freedom the stationary population of levels is determined by the probability of radiation in the ADE and NDE regions. The possibility of doing population inversion by means of ADE has been tackled in the Russian literature since long ago. A quantum system with many levels propagating superluminally in a medium has been discussed for the first time by Ginzburg and Fain in 1958 [61]. The inverse population of levels by means of ADE or a combination of ADE and acceleration may be enhanced whenever the ADE region is made larger than the NDE region. This is possible, e.g., in a medium with a big index of refraction. Naryshkina [62] already found in 1962 that the radiation of longitudinal waves in the ADE region is always greater than in the NDE region. However, her work remained unnoticed until 1984, when Nemtsov [63] wrote a short note on the advantage of using ADE longitudinal waves to invert a quantum system propagating in an isotropic plasma. The same year, Nemtsov and Eidman [64] demonstrated inverse population by ADE for the Landau levels of an electron beam propagating in a medium to which a constant magnetic field is applied. More recently, Kurian, Pirojenko and Frolov [65] have shown that in certain conditions (for certain range of the parameters), a detector moving with constant superluminous velocity on a circular trajectory inside a medium may be inverted too. Bolotovsky and Bykov [66] have studied the space-time properties of ADE on the simple case of a superluminous dipole propagating in uniform rectilinear motion in a nondispersive medium. These authors are positive with the separate observation of the ADE phenomenon for this case.

The radiation of a uniformly moving superluminal neutral polarizable particle has been studied by Meyer [67]. Frolov and Ginzburg [60] remarked that this case is an analog of ADE due to zero-point fluctuations of electric polarizability.

Moreover, we can modify the index of refraction in the Doppler factor in such a manner as to get the ADE conditions already at sublight velocities. In this way a more direct link to the Unruh effect is available, as has been shown also by Brevik and Kolbenstvedt[68]. These authors studied in detail the DeWitt detector moving through a dielectric nondispersive medium with constant velocity as well as with constant acceleration. In this way, first order perturbation theory formulas for transition probabilities and rates of emitted energy were exploited.

Negative energy waves as induced ADE processes

Let us mention here that one way to look at negative energy waves in plasma physics is to consider them as a manifestation of induced ADE elementary events discussed in the book of Nezlin [69]. As a matter of fact, a number of authors have already dealt with the problem of amplification and generation of electromagnetic waves based on ADE in the field of quantum electronics [70]. For details on the nonlinear instabilities in plasmas related to the existence of linear negative energy perturbations expressed in terms of specific creation and annihilation operators, and also for a discussion of the complete solution of the three-oscillator case with Cherry-like nonlinear coupling, one should consult the Trieste series of lectures delivered by Pfirsch [71].

ADE elementary processes for channeled particles

Baryshevskii and Dubovskaya [72] considered ADE processes for channeled positrons and electrons.

Negative energy waves in astrophysics/cosmology

Kandrup and O'Neill [73] investigated the hamiltonian structure of the Vlasov-Maxwell system in curved background spacetime with ADM splitting into space plus time, showing the importance of negative energy modes for time-independent equilibria.

3.2. Unruh Effect and Hadron Physics

The first application of the Unruh effect concept to hadron physics belongs to Barshay and Troost [74] in 1978. These authors identified the hadronic Hagedorn temperature with the Unruh temperature. In the same period, Hosoya [75] and also Horibe [76] applied moving mirror models to the thermal gluon production. An estimation of the contribution of Unruh's effect to soft photon production by quarks is given in [77]. Dey *et al* [78] related the Unruh temperature to the observed departure from the Gottfried sum rule for the difference of the proton and neutron structure functions in deep inelastic electron scattering. Also, Parentani and Potting [79] commented on the relationship between the limiting Hagedorn temperature the maximal acceleration and the Hawking temperature.

3.3. Decay of Accelerated Protons

The stability of protons has been long used as a basic test for the standard model of elementary particles. The most recent high-precision experiment set $\tau_p > 1.6 \cdot 10^{25}$ years [80] higher than the present age of the universe. However, long ago Ginzburg and Syrovatskii [81] speculated on the possibilty of decay in the case of noninertially moving protons.

Recently, Vanzella and Matsas have calculated in the context of *standard QFT* (in inertial frames) the weak interaction decay rate for uniformly accelerated protons [82]:

$$p^+ \rightarrow n^0 + e_M^+ + \nu_M \tag{93}$$

and shown that in certain astrophysical situations the proton lifetime may be quite short. The energy necessary to render the process (93) possible is supplied by the external accelerating agent. The tree-level formula for the proton proper lifetime in 1+1 spacetime dimensions [83] (valid under the no-recoil condition $a \ll m_p$) is

$$\tau_{\text{Inert}}^{p \rightarrow n}(a) = \frac{2\pi^{3/2} e^{\pi \frac{\Delta m}{a}}}{G_F^2 m_e} \left[G_{1\,3}^{3\,0} \left(\frac{m_e^2}{a^2} \,\middle|\, \begin{array}{c} 1 \\ -\frac{1}{2}\,,\ \frac{1}{2}+i\frac{\Delta m}{a}\,,\ \frac{1}{2}-i\frac{\Delta m}{a} \end{array} \right) \right]^{-1}, \tag{94}$$

where $G_{p\,q}^{mn}$ is the Meijer function [84], a is the proton proper acceleration, $\Delta m \equiv m_n - m_p$ and $m_\nu = 0$ is assumed. (Here m_p, m_n, m_e and m_ν are the rest masses of the proton, neutron, electron and neutrino, respectively.) The effective Fermi constant is $G_F = 9.9 \times 10^{-13}$ is determined from phenomenology.

Since a uniformly accelerated proton can be confined in a Rindler wedge which is a globally hyperbolic spacetime possessing a global timelike isometry, the associated uniformly accelerated observers (Rindler observers) must be able to analyze this phenomenon and reobtain the same (scalar) value for the proton lifetime (94) obtained with standard QFT. Notwithstanding, because of energy conservation, Rindler observers would simply claim that protons are precluded from decaying into a neutron through

$$p^+ \rightarrow n^0 + e_R^+ + \nu_R . \tag{95}$$

If Unruh's thermal bath (Minkowski vacuum) is taken into account new channels are opened:

$$p^+ + e_R^- \rightarrow n^0 + \nu_R \tag{96}$$

$$p^+ + \bar{\nu}_R \rightarrow n^0 + e_R^+ \tag{97}$$

$$p^+ + \bar{\nu}_R + e_R^- \rightarrow n^0 , \tag{98}$$

i.e., from the point of view of the Rindler observers, the proton should be transformed into a neutron through the *absorption* of a Rindler electron and/or anti-neutrino from the surrounding thermal bath providing the necessary energy to allow the process to occur. Any energy in excess can eventually be disposed by the emission of a neutrino or a positron (depending on the case). Performing an independent QFT calculation in the uniformly accelerated frame, Vanzella and Matsas obtained the following proper lifetime for the proton, after combining (incoherently) processes (96)-(98) in the presence of the vacuum thermal bath:

$$\tau_{\text{Rindler}}^{p \rightarrow n} = \frac{\pi^2 a e^{\pi \frac{\Delta m}{a}}}{G_F^2 m_e} \left[\int_{-\infty}^{+\infty} d\omega_R \frac{K_{i\frac{\omega_R}{a}+\frac{1}{2}}(m_e/a) K_{i\frac{\omega_R}{a}-\frac{1}{2}}(m_e/a)}{\cosh[\pi(\omega_R - \Delta m)/a]} \right]^{-1} . \tag{99}$$

Although Eqs. (94) and (99) appear to be quite different, Vanzella and Matsas have shown numerically that they coincide up to the level [85]:

$$\Delta^{-1} \int_\Delta [(\tau_{\text{Rindler}}^{p \rightarrow n} - \tau_{\text{Inert}}^{p \rightarrow n})/\tau_{\text{Rindler}}^{p \rightarrow n}]^2 dx \sim 10^{-16},$$

where $x \equiv \log_{10}(a/1MeV)$.

Acknowledgements

[1] S.J. Jasper, Bull. École Polytech. Jassy **2**, 262 (1947)

[2] J.L. Synge, Proc. Roy. Irish Acad. A **65**, 27 (1967)

[3] E. Honig, E.L. Schucking, C.V. Vishveshwara, J. Math. Phys. **15**, 774 (1974)

[4] J.R. Letaw, Phys. Rev. D **23**, 1709 (1981); J.D. Pfautsch, "Quantum Field Theory in Stationary Coordinate Systems", Ph.D. Thesis, Univ. of Texas at Austin (1981)

[5] B. R. Iyer, C.V. Vishveshwara, Class. Quantum Grav. **5**, 961 (1988); Phys. Rev. D **48**, 5706 (1993), gr-qc/9310019

[6] D. Bini, R.T. Jantzen, A. Merloni, Class. Quantum Grav. **16**, 1 (1999)

[7] A.W. Chao, *Accelerator Physics* (World Scientific, 2000)

[8] S.K. Kim, K.S. Soh, and J.H. Yee, Phys. Rev. D **35**, 557 (1987)

[9] J.R. Letaw and J.D. Pfautsch, Phys. Rev. D **22**, 1345 (1980)

[10] H.C. Rosu, Gravitation & Cosmology **7**, 1 (2001), gr-qc/9406012

[11] H.C. Rosu, in *Artificial Black Holes*, pp 307-334 (World Scientific, 2002), gr-qc/0012083

[12] V.I. Man'ko and R. Vilela Mendes, Phys. Lett. A **263**, 53 (1999) [physics/9712022].

[13] P. Candelas and D.W. Sciama, Phys. Rev. Lett. **38**, 1372 (1977).

[14] L.F. Cugliandolo, J. Kurchan and L. Peliti, Phys. Rev. E **55**, 3898 (1997) [cond-mat/9611044]; R. Exartier and L. Peliti, Eur. Phys. J. **B 16**, 119 (2000), [cond-mat/9910412].

[15] B.L. Hu and P.R. Johnson, in QABP-2 (World Scientific, 2002), quant-ph/0012132

[16] J.S. Bell, J.M. Leinaas, Nucl. Phys. B **212**, 131 (1983)

[17] J.S. Bell, R.J. Hughes, J.M. Leinaas, Z. Phys. C **28**, 75 (1985)

[18] J.S. Bell, J.M. Leinaas, Nucl. Phys. B **284**, 488 (1987)

[19] K.T. McDonald, in Proc. 1987 IEEE Part. Accel. Conf. vol. 2, 1196 (1987), pp. 1196-1197, physics/0003060

[20] J. Leinaas, Europhysics News **22**, 78 (1991)

[21] A.A. Sokolov and I.M. Ternov, Dokl. Akad. Nauk. SSSR **153**, 1052-1054 (1963) [Sov. Phys. Dokl. **8**, 1203 ((1964)]

[22] Ya.S. Derbenev and A.M. Kondratenko, Sov. Phys. JETP **37**, 968 (1973)

[23] D.P. Barber and S.R. Mane, Phys.Rev. A **37**, 456 (1988)

[24] J.R. Johnson, R. Prepost, D.E. Wiser, J.J. Murray, R.F. Schwitters, C.K. Sinclair, Nucl. Instr. and Methods **204**, 261 (1983)

[25] Y.Q. Cai, D.G. Lloyd, and G. Papini, Phys. Lett. A **178**, 225 (1993)

[26] E. Bautista, Phys. Rev. D **48**, 783 (1993)

[27] H. Rosu, Nuovo Cimento B **109**, 423 (1994) and also gr-qc/9412033

[28] M. Kühne, in *New Developments and Applications in Optical Radiometry*, Inst. Phys. Conf. Ser. No. 92, IOP Publishing Ltd, (1989)

[29] A. Klein and L.J. Landau, J. Func. Analysis **42**, 368 (1981)

[30] K. Kirsebom et al., ::: Phys. Rev. Lett. **87**, 054801 (2001). For theory, see A.V. Korol, A.V. Solov'yov and W. Greiner, J. Phys. G: Nucl. Part. Phys. **28**, 627 (2002)

[31] U. Uggerhoj, private communication at QABP-3.

[32] J. Rogers, Phys. Rev. Lett. **61**, 2113 (1988)

[33] L.S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986)

[34] H. Rosu, Nuovo Cimento B **111**, 507 (1996)

[35] O. Levin, Y. Peleg, and A. Peres, J. Phys. A: Math. Gen. **26**, 3001 (1993)

[36] J. Tan and G. Gabrielse, Appl. Phys. Lett. **55**, 2144 (1989); Phys. Rev. A **48**, 3105 (1993)

[37] E.Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989)

[38] W. Cochran, Am. J. Phys. **57**, 1039 (1989)

[39] V. Frolov and I. Novikov, Phys. Rev. D **48**, 4545 (1993)

[40] V.V. Dodonov, A.B. Klimov, and D.E. Nikonov, Phys. Rev. A **47**, 4422 (1993)

[41] G.T. Moore, J. Math. Phys. **11**, 2679 (1970)

[42] V.V. Hizhnyakov, Quantum Opt. **4**, 277 (1992)

[43] C.K. Law, Phys. Rev. A **49**, 433 (1994)

[44] C. Eberlein, Phys. Rev. Lett. **76**, 3842 (1996); Phys. Rev. A **53**, 2772 (1996)

[45] K.A. Milton, quant-ph/9608003

[46] L. Grishchuk, H.A. Haus, and K. Bergman, Phys. Rev. D **46**, 1440 (1992)

[47] S.M. Darbinian, K.A. Ispirian, A.T. Margarian, preprint Yerevan Phys.Inst. YERPHY-1188(65)-89 (August 1989)

[48] B. Dehning, A.C. Melissinos, F. Perrone, C. Rizzo, G. von Holtey, Phys. Lett. B **249**, 145 (1990)

[49] C. Bini et al., Phys. Lett. B **262**, 135 (1991). For a more recent work, see L.S. Brown and R.S. Steinke, "Compton scattering on black body photons", hep-ph/9606303.

[50] Unauthored, "LEP Takes its Temperature", CERN Courier **31**, 2 (March 1991)

[51] S.M. Darbinian, K.A. Ispiryan, M.K. Ispiryan, A.T. Margaryan, JETP Lett. **51**, 110 (1990)

[52] P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999)

[53] H. Rosu, Physics World **12**, 21 (October 1999), physics/9910015

[54] M. Visser, in Matters of Gravity **17** (Spring 2001), gr-qc/0102044

[55] K.T. McDonald, "The Hawking-Unruh temperature and damping in a linear focusing channel", in QABP-1 (World Scientific, 1999), physics/0003061

[56] W.G. Unruh and R.M. Wald, Phys. Rev. D **29**, 1047 (1984); A. Higuchi, G.E.A. Matsas, and D. Sudarsky, Phys. Rev. D **46**, 3450 (1992) and D **45**, R3308 (1992)

[57] V.L. Ginzburg and I.M. Frank, Dokl. Akad. Nauk **56**, 583 (1947)

[58] I.M. Frank, Usp. Fiz. Nauk **129**, 685 (1979); *Vavilov- Cherenkov Radiation. Theoretical Aspects* (Nauka, Moskow, 1988)

[59] V.L. Ginzburg, in *Progress in Optics XXXII*, Ed. E. Wolf (Elsevier, 1993) [Russian version in FIAN **176** (Nauka, Moskow, 1986)]. The 1993 version is updated and has a new section with comments on acceleration radiation.

[60] V.P. Frolov and V.L. Ginzburg, Phys. Lett. A **116**, 423 (1986)

[61] V.L. Ginzburg and V.M. Fain, JETP **8**, 567 (1959) [JETF **35**, 817 (1958)]

[62] L.G. Naryshkina, JETF **43**, 953 (1962)

[63] B.E. Nemtsov, Pis'ma v JTF **10**, 588 (1984)

[64] B.E. Nemtsov and V. Ya. Eidman, JETF **87**, 1192 (1984) [Sov. Phys. JETP **60**, 682 (1984)]

[65] V.E. Kurian, A.V. Pirojenko, V.P. Florov, "On the Possibility of Population Inversion in Quantum Systems Moving Uniformly on a Circle in a Medium", FIAN preprint 142 (1988); KSF **10**, 54 (1988)

[66] B.M. Bolotovski and V.P. Bykov, Radiofizika **32**, 386 (1989)

[67] P.P. Meyer, J. Phys. A: Math. Gen. **18**, 2235 (1985)

[68] I. Brevik and H. Kolbenstvedt, Nuovo Cimento B **103**, 45 (1989)

[69] M.V. Nezlin, *Dynamics of Beams in Plasmas*, in Russian, (Energoizdat, 1982)

[70] N.S. Ginzburg, Radiofizika **22**, 470 (1979); A.N. Didenko et al., Pis'ma v JTF **9**, 1207 (1983); M.V. Kuzelyev and A.A. Rukhadze, Fizika Plasmy **15**, 1122 (1989)

[71] D. Pfirsch, "Negative-energy Modes in Collisionless Kinetik Theories and their Possible Relation to Nonlinear Instabilities", series of lectures at the Plasma College, ICTP-Trieste (June 1991)

[72] V.G. Baryshevskii and I. Ya. Dubovskaya, Sov. Phys. Dokl. **21**, 741 (1976)

[73] H.E. Kandrup and E. O'Neill, Phys. Rev. D **48**, 4534 (1993)

[74] S. Barshay and W. Troost, Phys. Lett. B **73**, 437 (1978)

[75] A. Hosoya, Prog. Theor. Phys. **61**, 280 (1979)

[76] M. Horibe, Prog. Theor. Phys. **61**, 661 (1979)

[77] S.M. Darbinian, K.A. Ispirian, A.T. Margarian, Sov. J. Nucl. Phys.**54**, 364 (1991) [Yad. Fiz. 54, 600 (1991)]

[78] J. Dey et al., Phys. Lett. A **172**, 203 (1993)

[79] R. Parentani and R. Potting, Phys. Rev. Lett. **63**, 945 (1989)

[80] C. Caso et al., Eur. Phys. J. **C** 3, 1 (1998)

[81] V.L. Ginzburg and S.I. Syrovatskii, Usp. Fiz. Nauk **87**, 65 (1965)

[82] D.A.T. Vanzella and G.E.A. Matsas, *Phys. Rev.* D **63**, 014010 (2001)

[83] G.E.A. Matsas and D.A.T. Vanzella, *Phys. Rev.* D **59**, 094004 (1999)

[84] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products* (1980, Academic Press, New York).

[85] D.A.T. Vanzella and G.E.A. Matsas, *Phys. Rev. Lett.* **87**, 151301 (2001)